Quantum circuits in cold atoms

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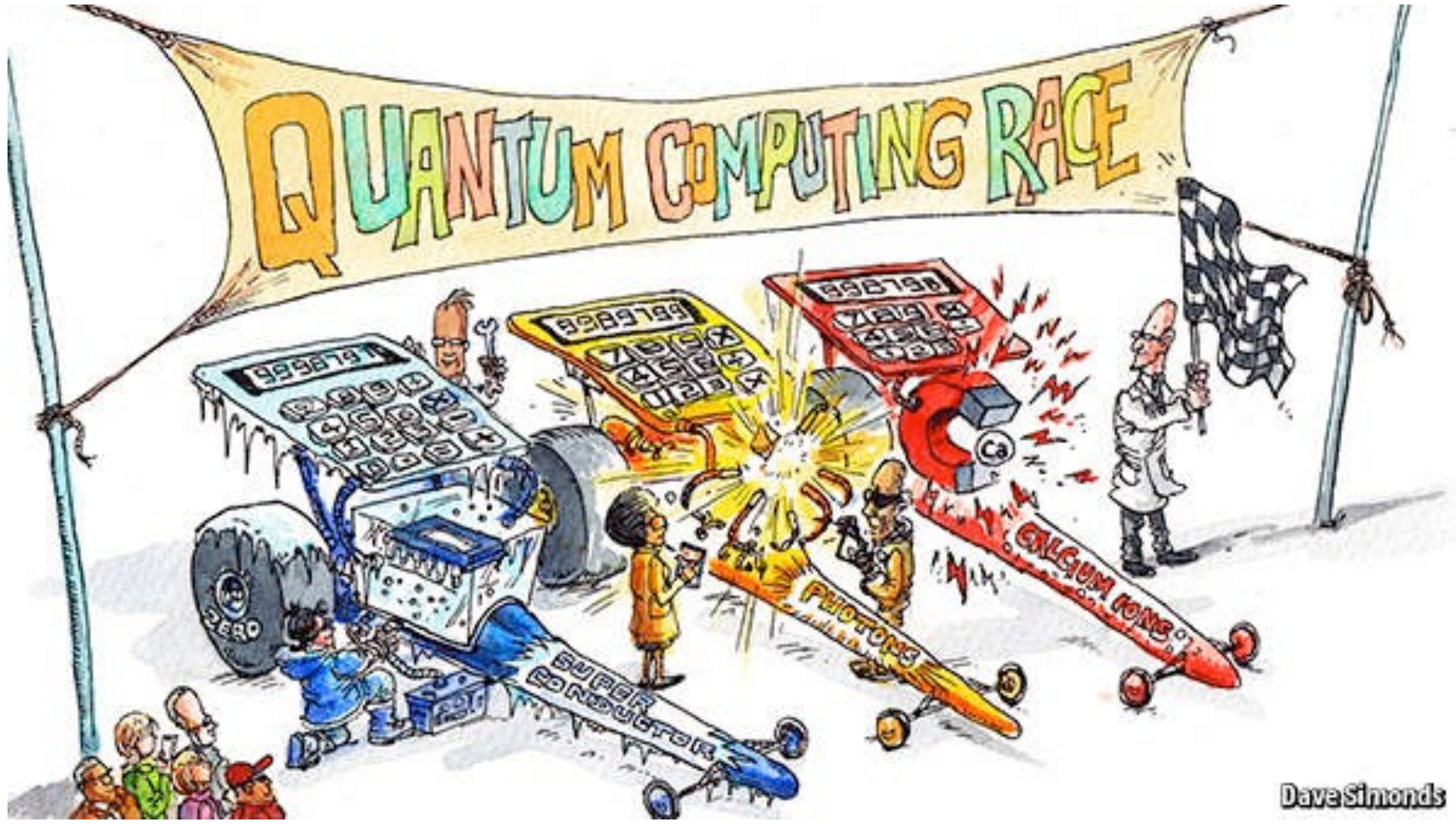


Superconducting Qubits

- Superconducting electrical components
- Excitations of superconductor forms qubit

Strengths

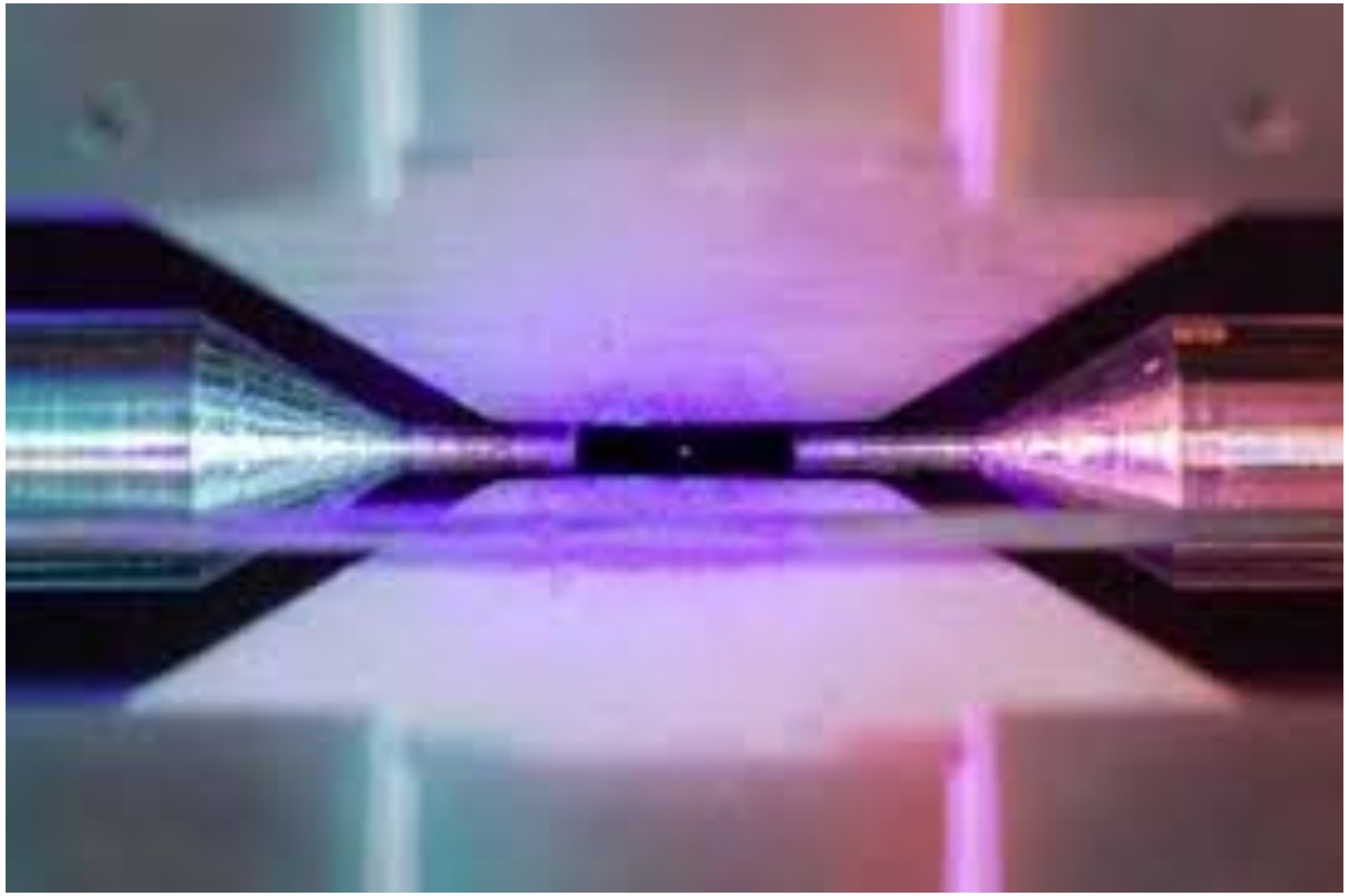
- Integrated in electrical circuit
- Control with GHz frequencies
- Great integration into software packages
- Large number of algorithms

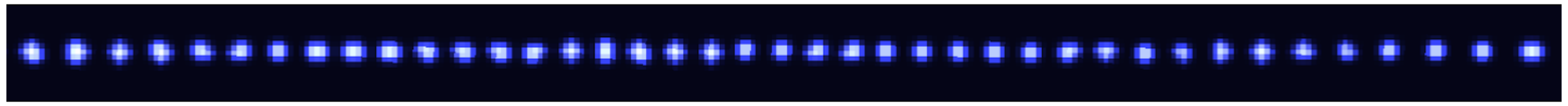
Open questions

- Size of system
- Fidelities
- Practical use

Trapped Ions

Lucas group, Oxford





Monroe group, JQI Maryland

- Charged particles
- Electrostatic traps
- Laser and microwave operations

Strengths

- Highest gate fidelities
- Fast experimental timescales

Open questions

- Maximum size 50 qubits so far
- Challenging scalability

Important commercial players

- IonQ (JQI)
- AQT (Innsbruck)
- Honeywell (JILA)

NYSE OPENING BELL®
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IONQ
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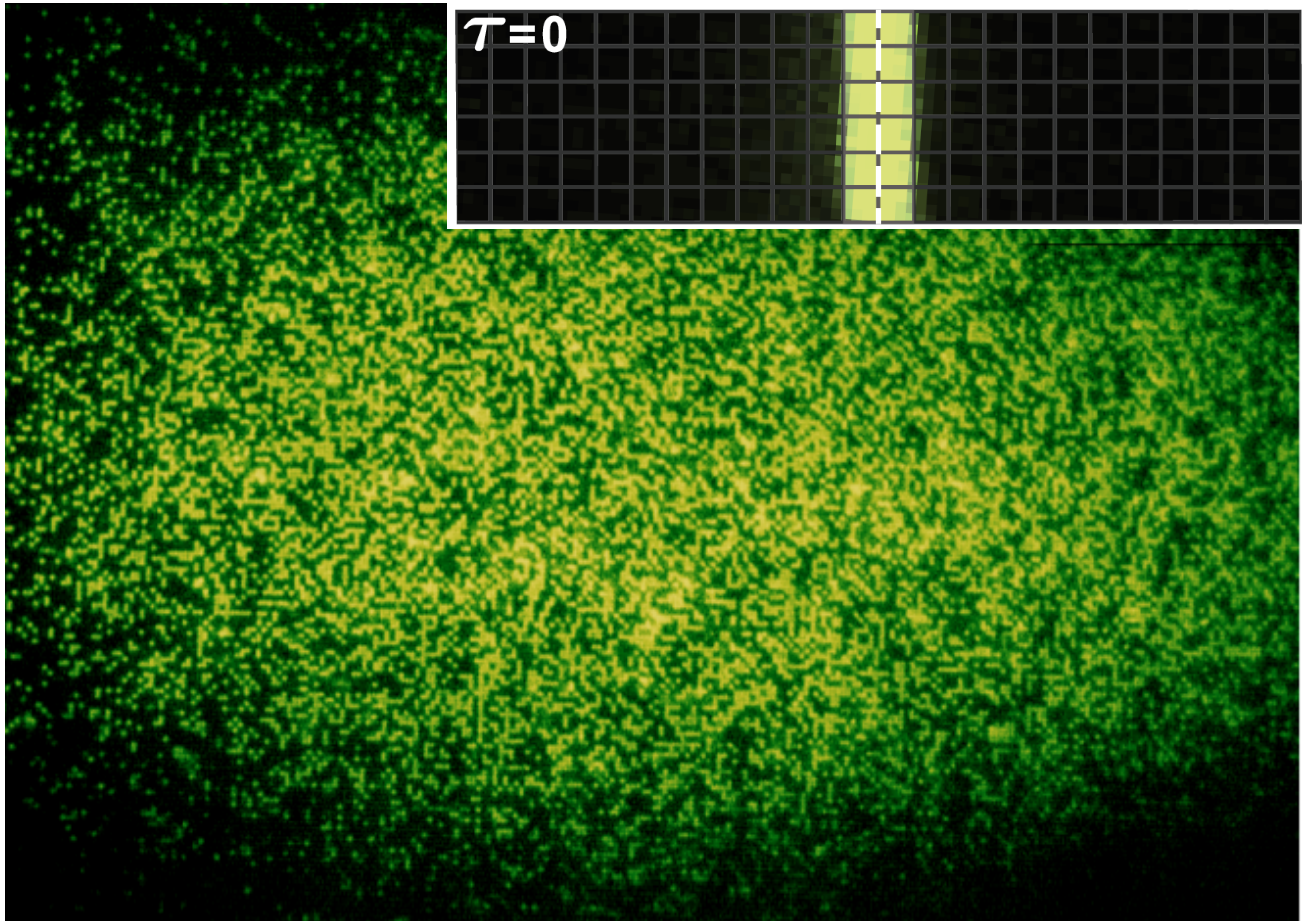
Neutral atoms

Greiner group, Harvard university



Lukin group, Harvard university

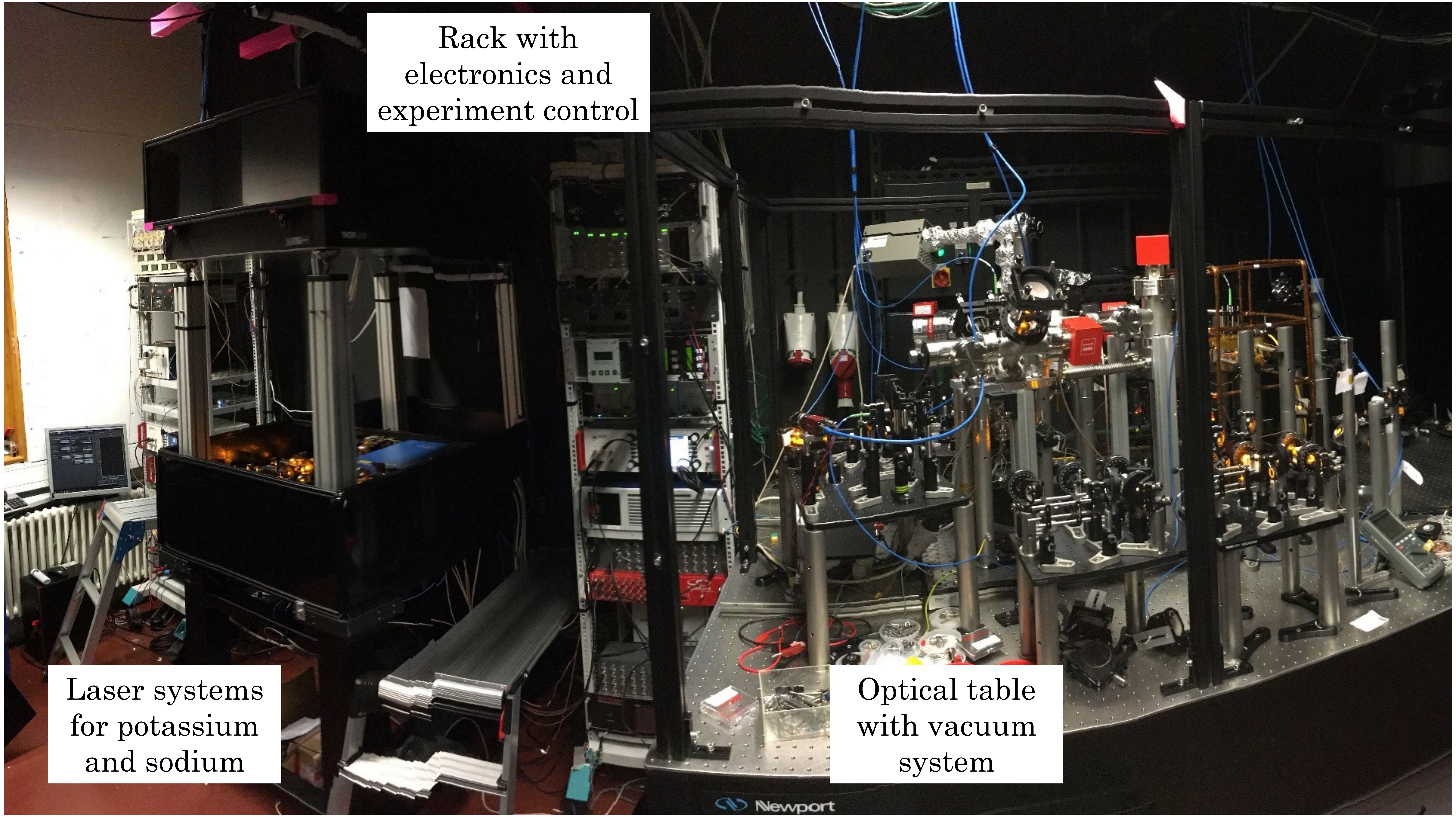
- Neutral laser-cooled particles
- Optical potentials
- Single-particle readout and control

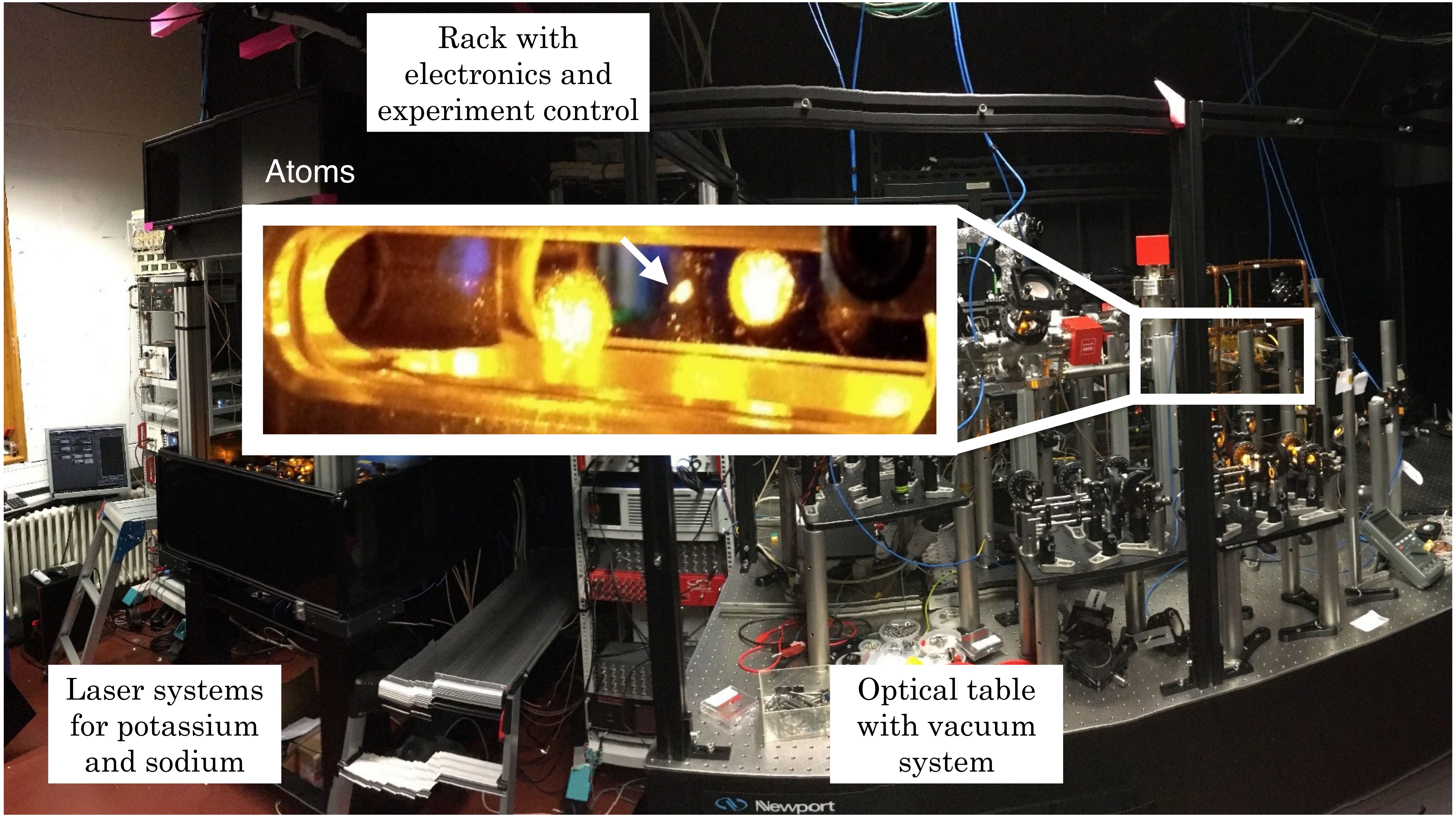
Strengths

- Many hundreds of particles
- Enormous flexibility

Open questions

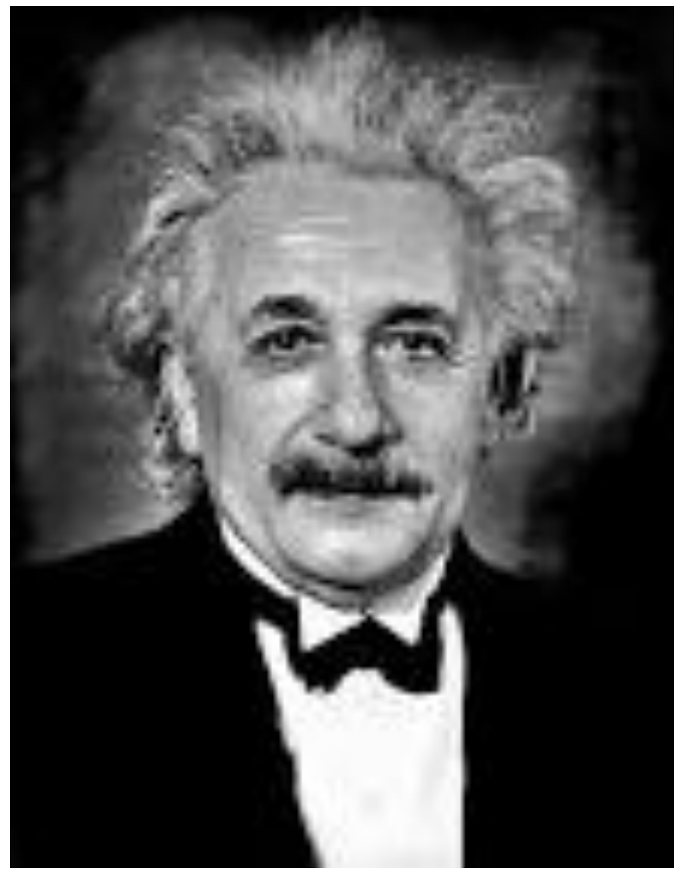
- Poor software integration
- few algorithms
- few applications outside of physics studied


Rack with
electronics and
experiment control
Laser systems
for potassium
and sodium
Optical table
with vacuum
system
Newport


Rack with electronics and experiment control
Atoms
Laser systems for potassium and sodium
Optical table with vacuum system
Newport

1. Atomic clocks — Qubits in cold atoms
2. Optical tweezers — Trapped qubits in atoms
3. Rydberg atoms — Large scale entanglement
4. Moving particles — Bosons vs Fermions and the link to chemistry
5. Lattice gauge theories — Working on a really hard physics problem

What is time ?



Einsteins' special relativity:

Time is what a clock measures.

Experimentalists dilemma: What is a clock ?

Something that ‚ticks', i.e. provides a regular series of events

Traditional clocks

1 tick = 1 day

1 tick = few seconds

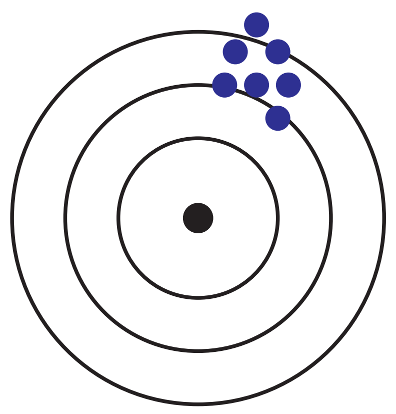
1 tick = 0.1 ms

<u>Problems:</u>

- Not very stable
- Very slow ticking
- Reproducility

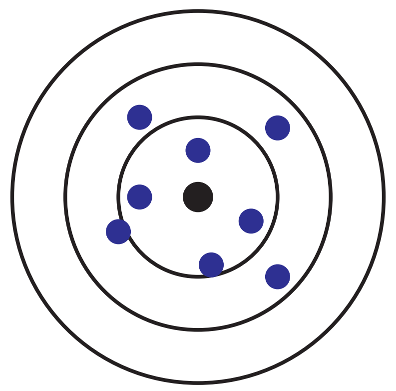
What is a good clock ?

Stable



repeat with the same clock lots of
measurements and get similar results

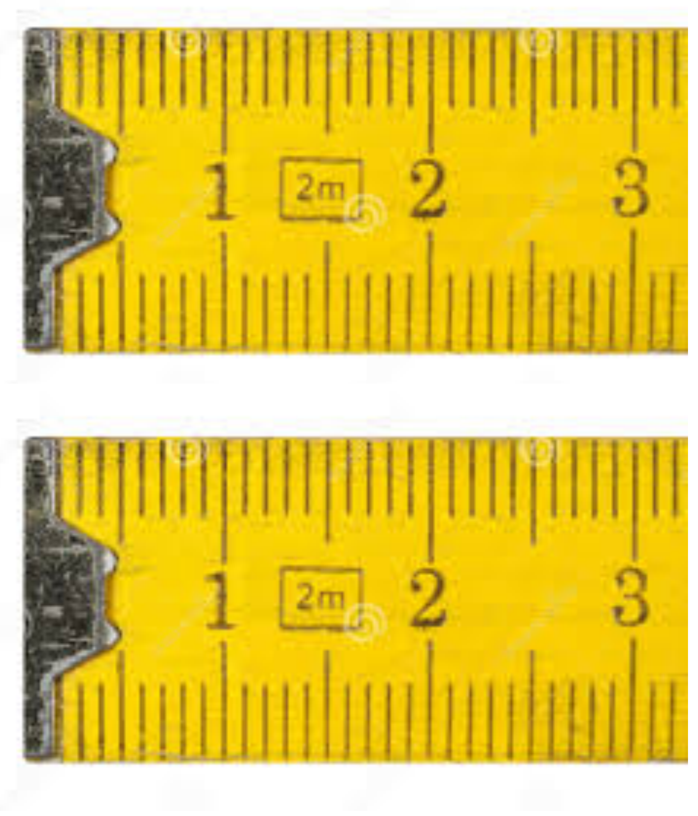
Precise

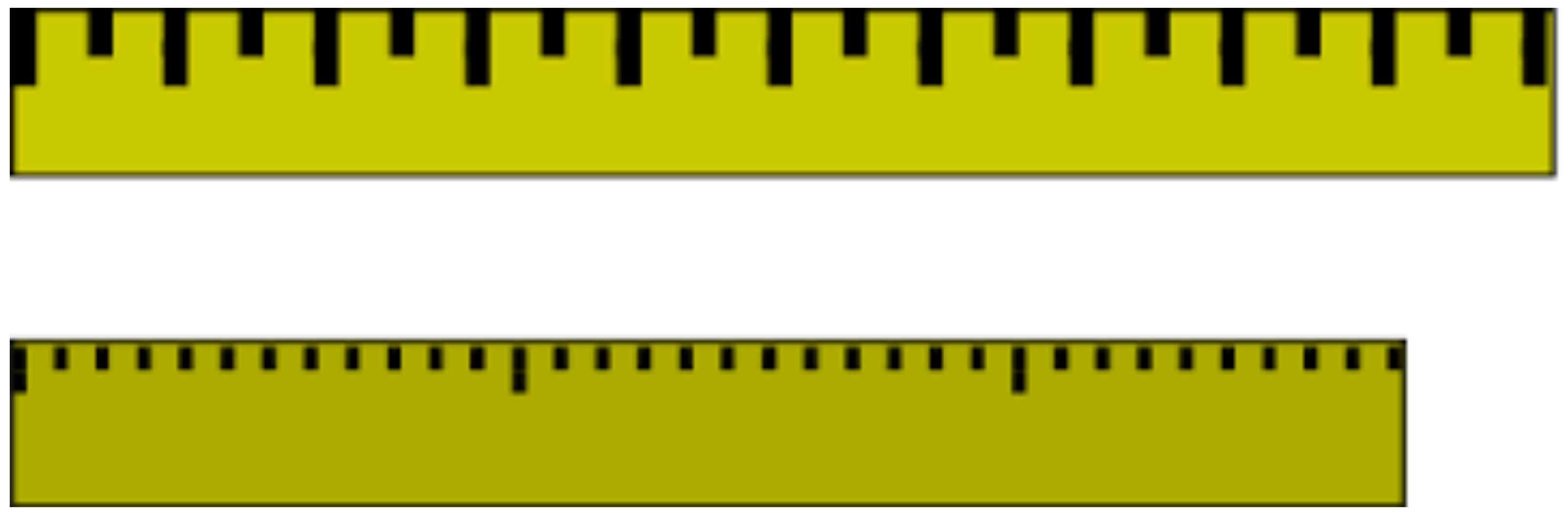


build several clocks
and obtain same results

most of the time much, much
harder to estimate

Characterization of clocks





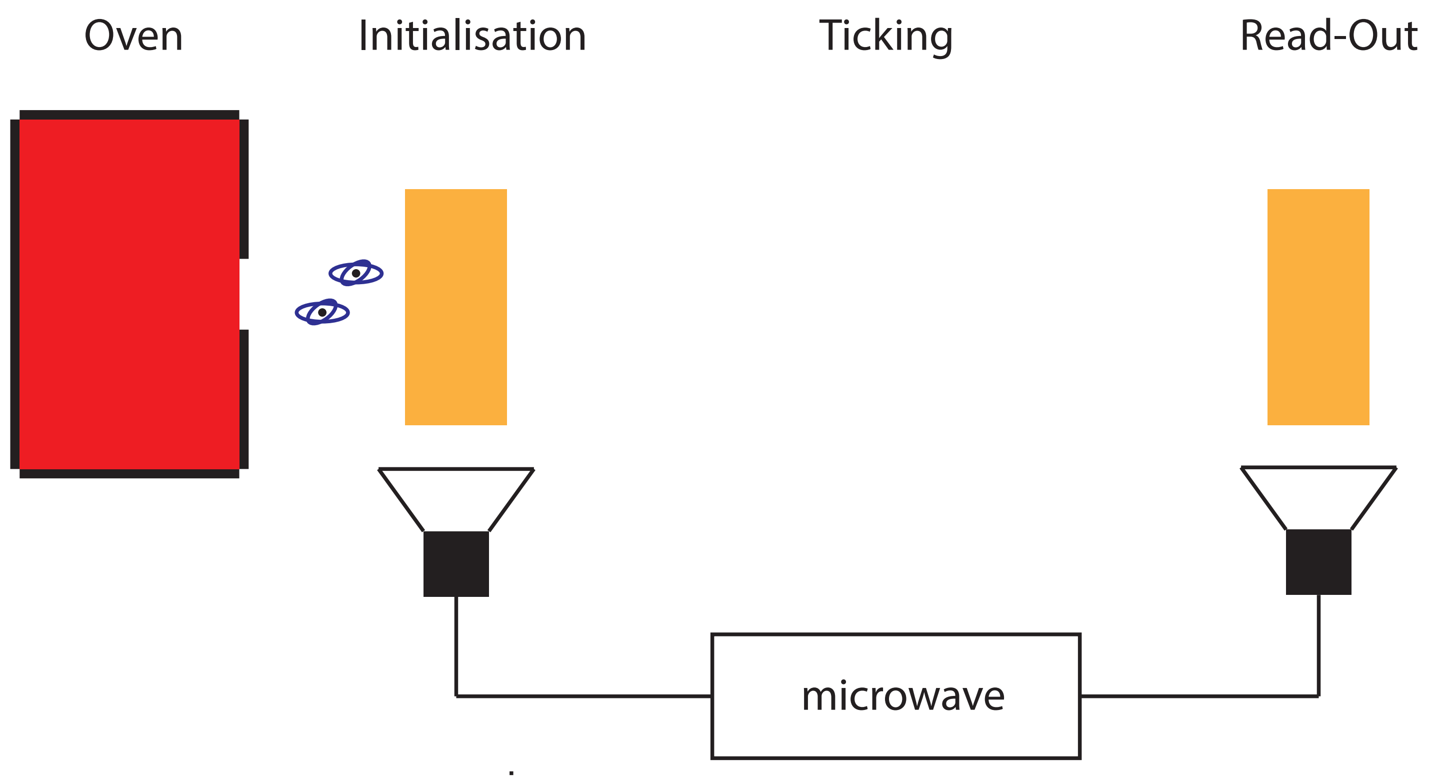
let them tick for a long time

compare the result

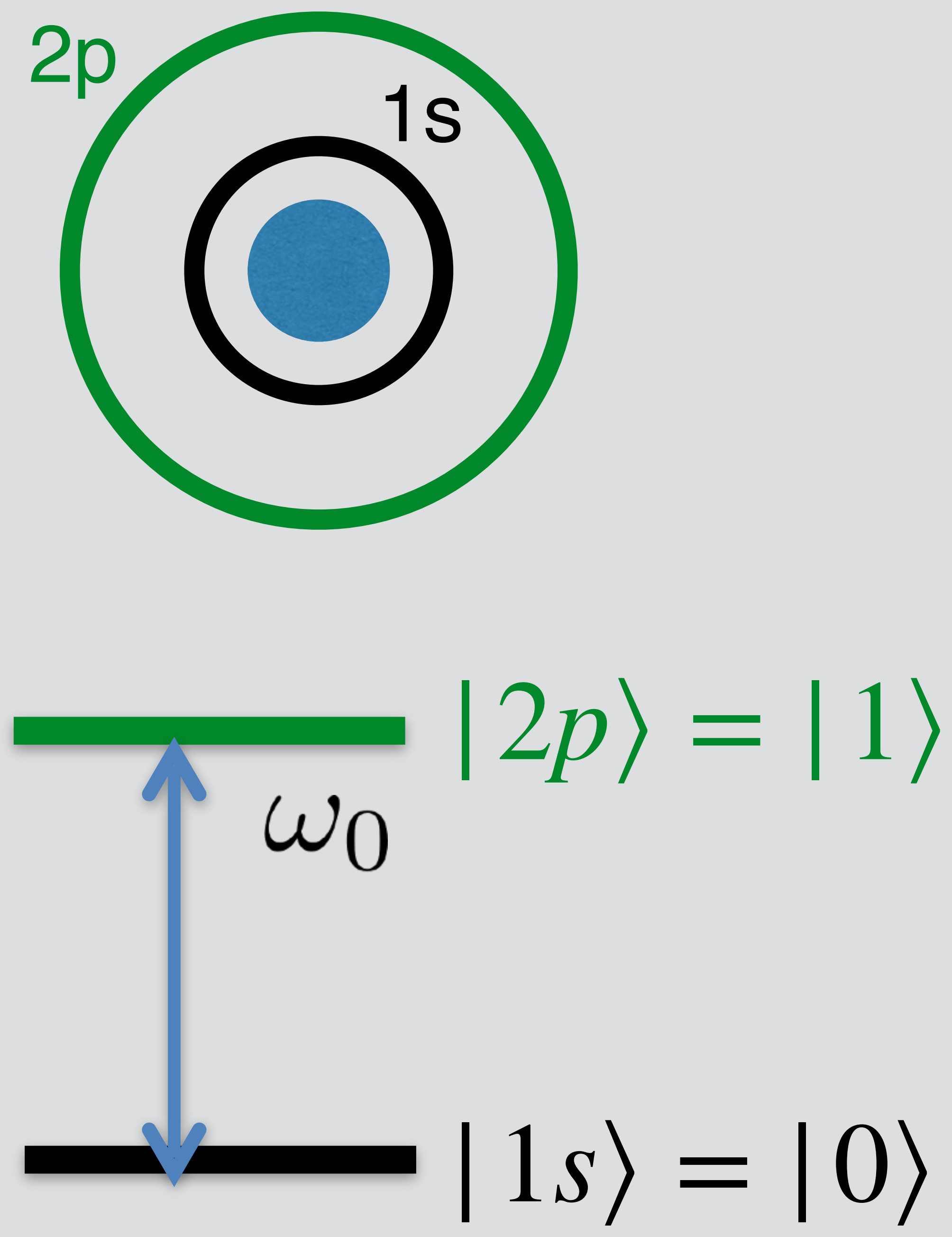
<u>What about precision?</u>

We need a good standard and atoms give this

Atomic clocks

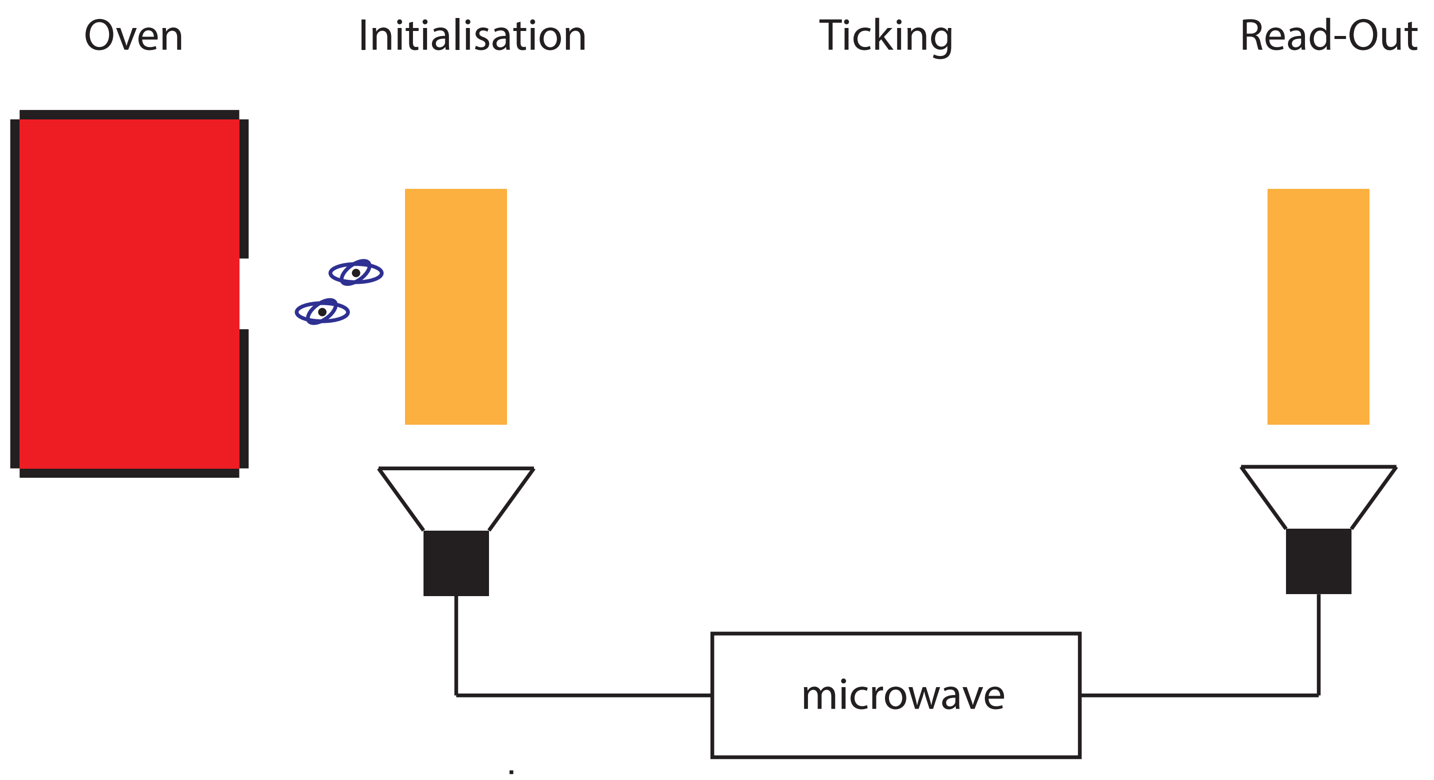


The Atom

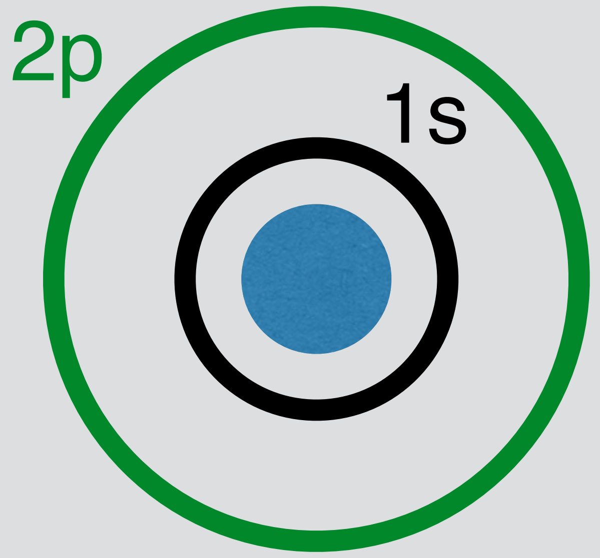


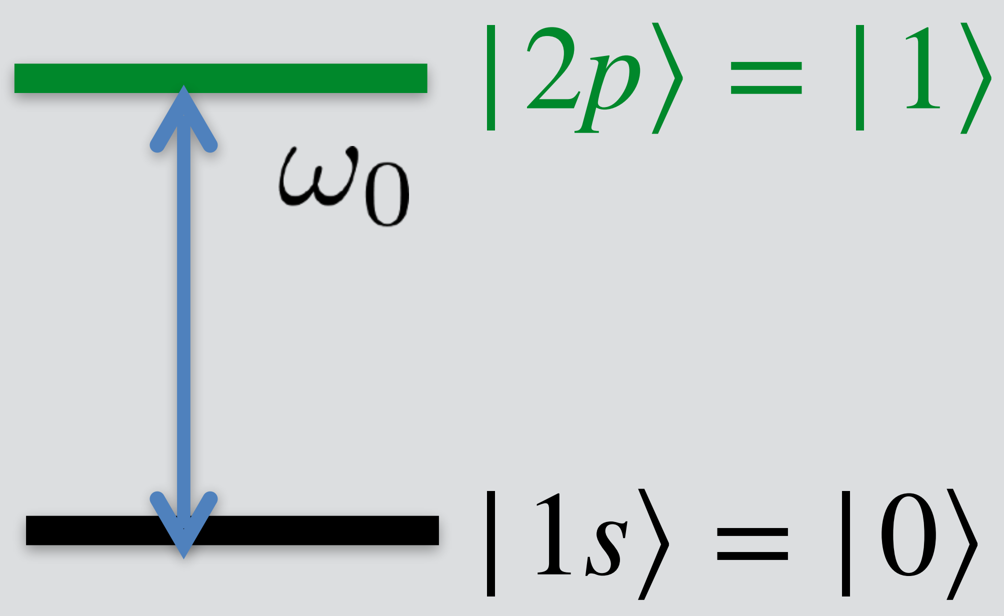
$$\mathcal{H} = E_0 |0\rangle\langle 0| + E_1 |1\rangle\langle 1|$$

Atomic clocks



The Atom

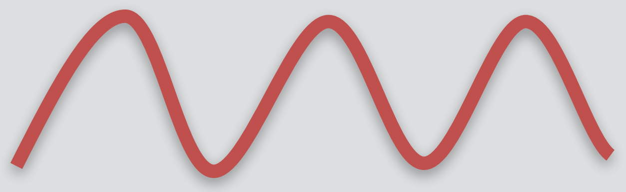




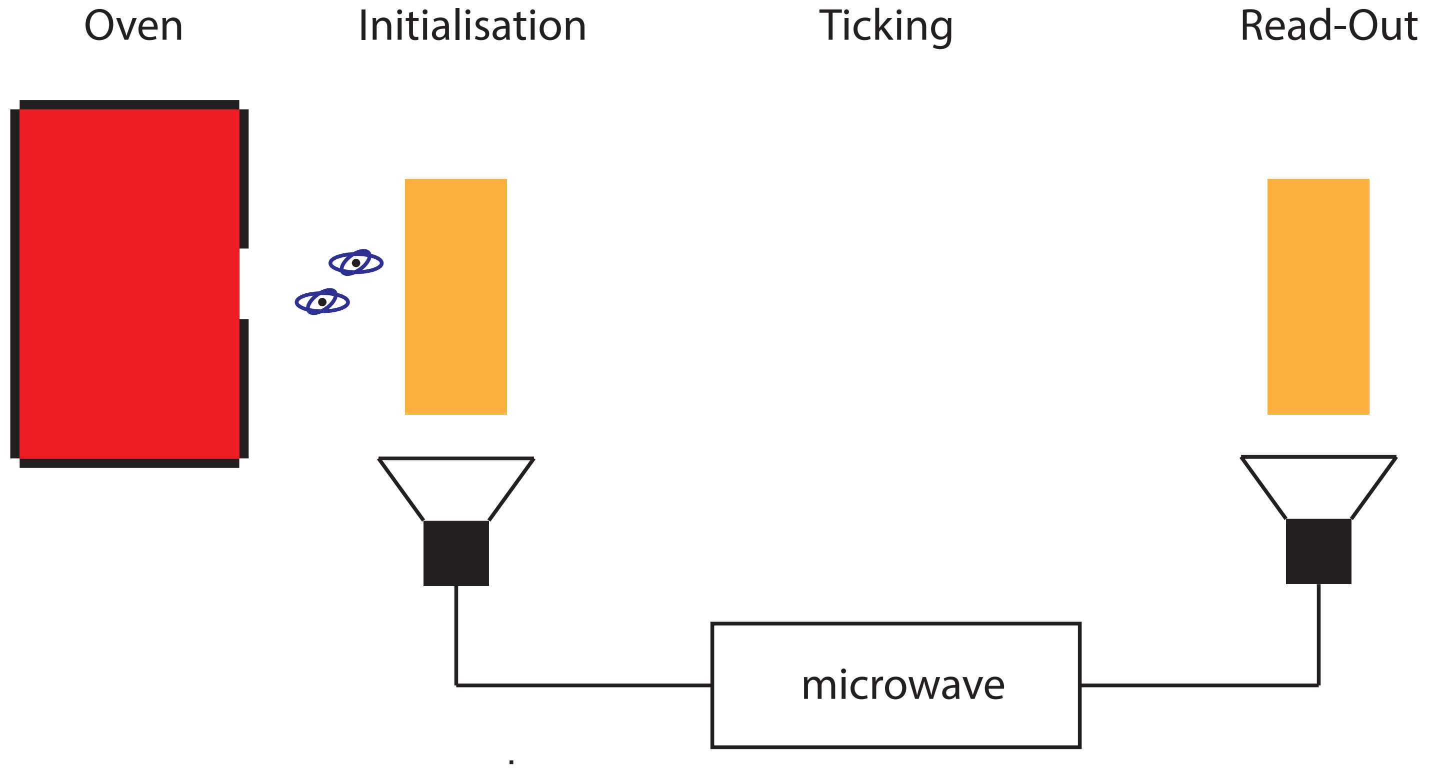
$$\mathscr{H} = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

The electric field

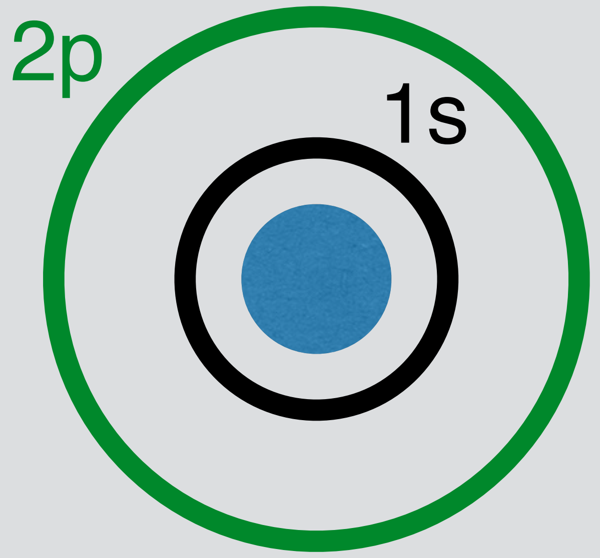
$$\mathbf{E} = E_0\left(e^{i\omega_L t + i\varphi} + e^{-i\omega_L t - i\varphi}\right)$$

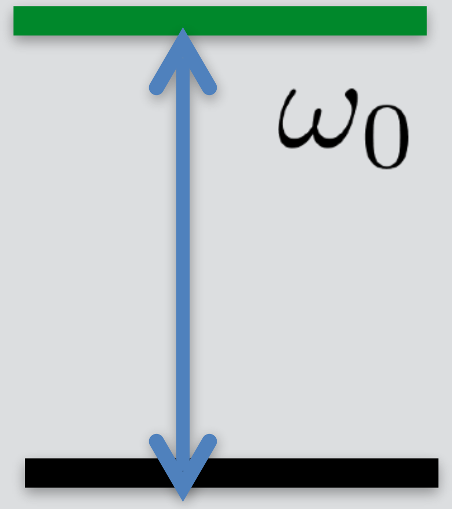


Atomic clocks



The Atom





$|2p\rangle = |1\rangle$

$|1s\rangle = |0\rangle$

$$\mathscr{H} = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

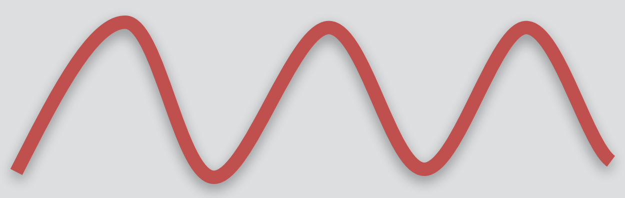
Interaction via

$$\mathcal{H} = -\mathbf{d}\cdot\mathbf{E}$$

$$\mathbf{d} = d\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)$$

The electric field

$$\mathbf{E} = E_0\left(e^{i\omega_L t + i\varphi} + e^{-i\omega_L t - i\varphi}\right)$$

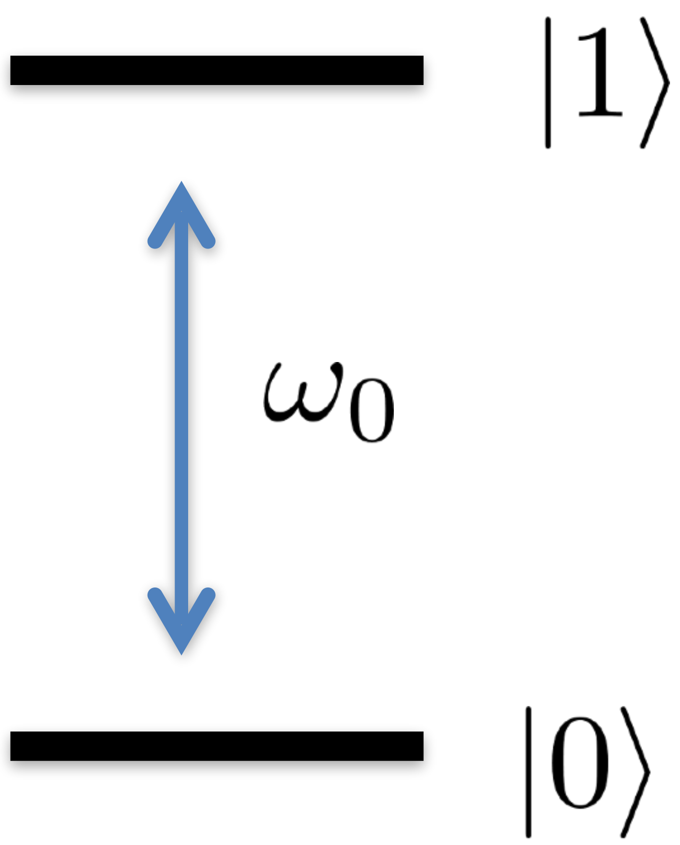


The atom as a qubit

$$\mathcal{H} = E_0 \left|0\right\rangle \left\langle 0\right| + E_1 \left|1\right\rangle \left\langle 1\right|$$

$$\mathcal{H} = \frac{\hbar\omega_0}{2} \left|1\right\rangle \left\langle 1\right| - \frac{\hbar\omega_0}{2} \left|0\right\rangle \left\langle 0\right|$$

$$\mathcal{H} = \frac{\hbar\omega_0}{2}\sigma_z$$



$\rightarrow$ write everything in terms of spins

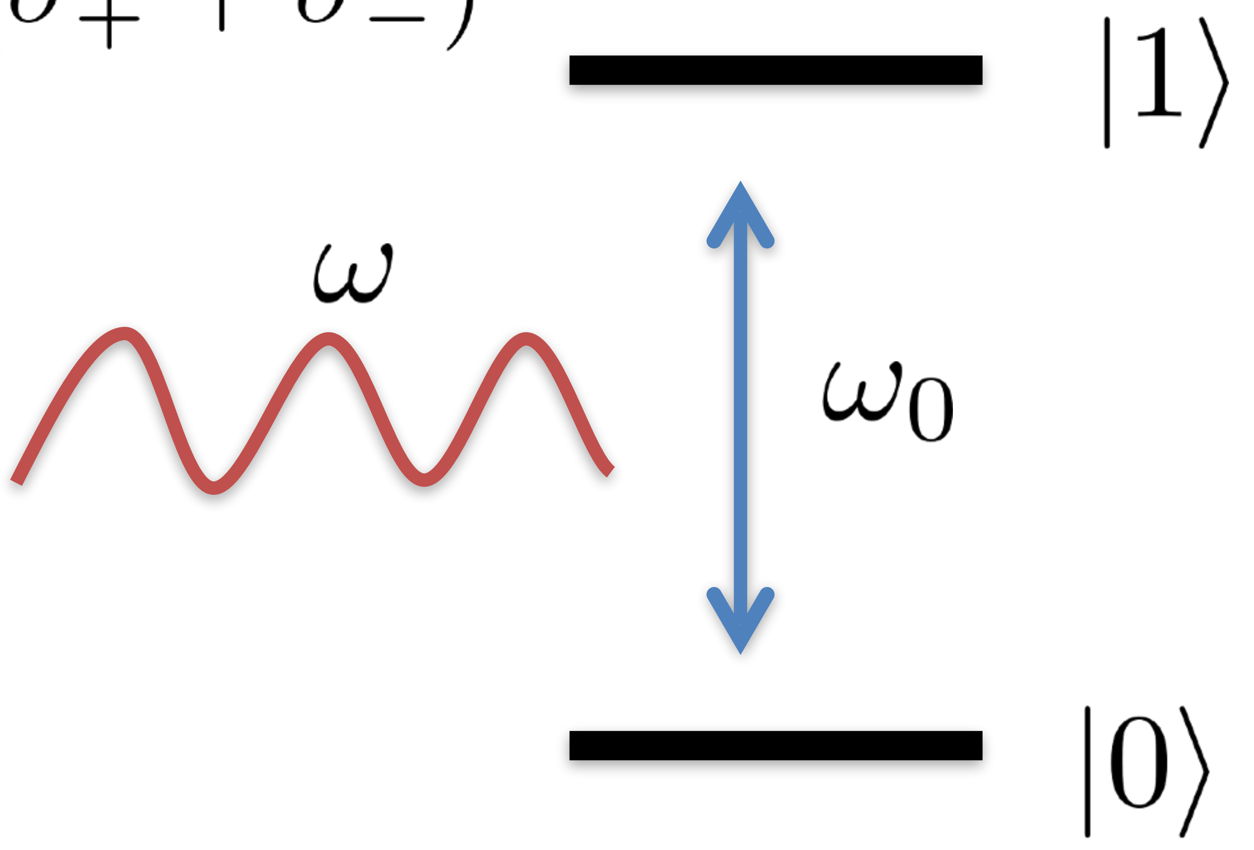
$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Interaction Hamiltonian

$$\mathcal{H} = -\mathbf{d}\cdot\mathbf{E}$$

$$\mathbf{E} = E(\mathrm{e}^{\mathrm{i}\omega t+\mathrm{i}\varphi} + \mathrm{e}^{-\mathrm{i}\omega t-\mathrm{i}\varphi})$$

$$\mathbf{d} = d(\sigma_+ + \sigma_-)$$



Rotating frame:

$$\mathcal{H} = \frac{dE}{2}(\sigma_+\mathrm{e}^{\mathrm{i}\varphi} + \sigma_-\mathrm{e}^{-\mathrm{i}\varphi})$$

$$\mathcal{H} \sim \hbar\Omega(\sigma_+\mathrm{e}^{\mathrm{i}\varphi} + \sigma_-\mathrm{e}^{-\mathrm{i}\varphi})$$

$\varphi = 0$

$$\mathcal{H} \sim \hbar\Omega(\sigma_+ + \sigma_-)$$

$$\mathcal{H} \sim \hbar\Omega\sigma_x$$

$\varphi = \pi/2$

$$\mathcal{H} \sim \hbar\Omega(\sigma_+ - \sigma_-)$$

$$\mathcal{H} \sim \hbar\Omega\sigma_y$$

Clocks as extremely precise qubits

Rotation about z-axis
Detuning

$$\mathcal{H} = \hbar\Delta\hat{\sigma}_z$$

$Z_{\pi/2}$ $\Delta t = \frac{\pi}{2}$

Rotation about x-axis
Laser intensity

$$\mathcal{H} = \hbar\Omega_x\hat{\sigma}_x$$

$X_{\pi/2}$ $\Omega_x t = \frac{\pi}{2}$

Rotation about y-axis
Laser intensity with phase adjusted

$$\mathcal{H} = \hbar\Omega_y\hat{\sigma}_y$$

$Y_{\pi/2}$ $\Omega_y t = \frac{\pi}{2}$

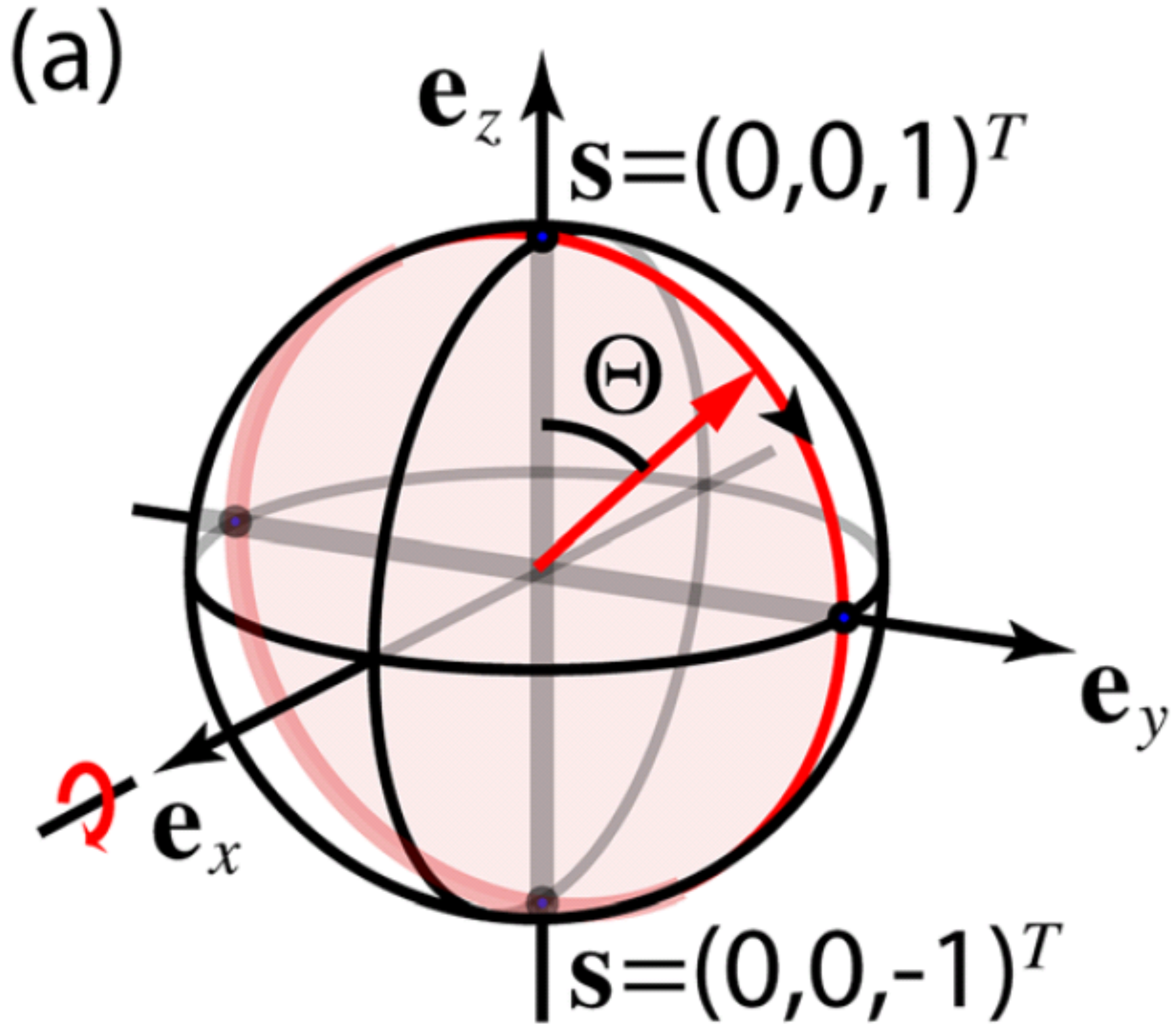
$$U = e^{i\mathcal{H}t/\hbar}$$

Example: Rabi oscillations

$\mathcal{H} = \hbar \Omega S_x$

time evolution: $e^{i\Omega \sigma_x t}$

Rotation about x-axis angle $\Theta = \Omega t$



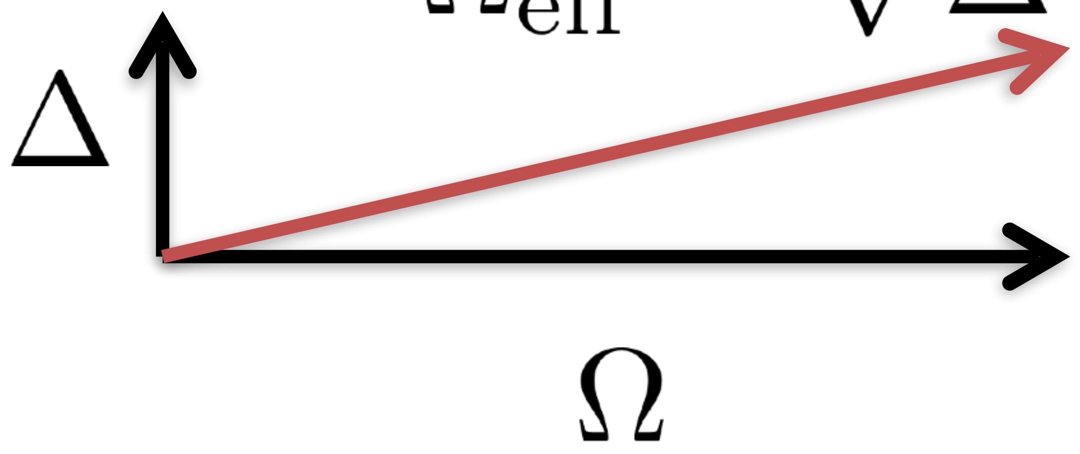
Example: Offresonant Rabi oscillations

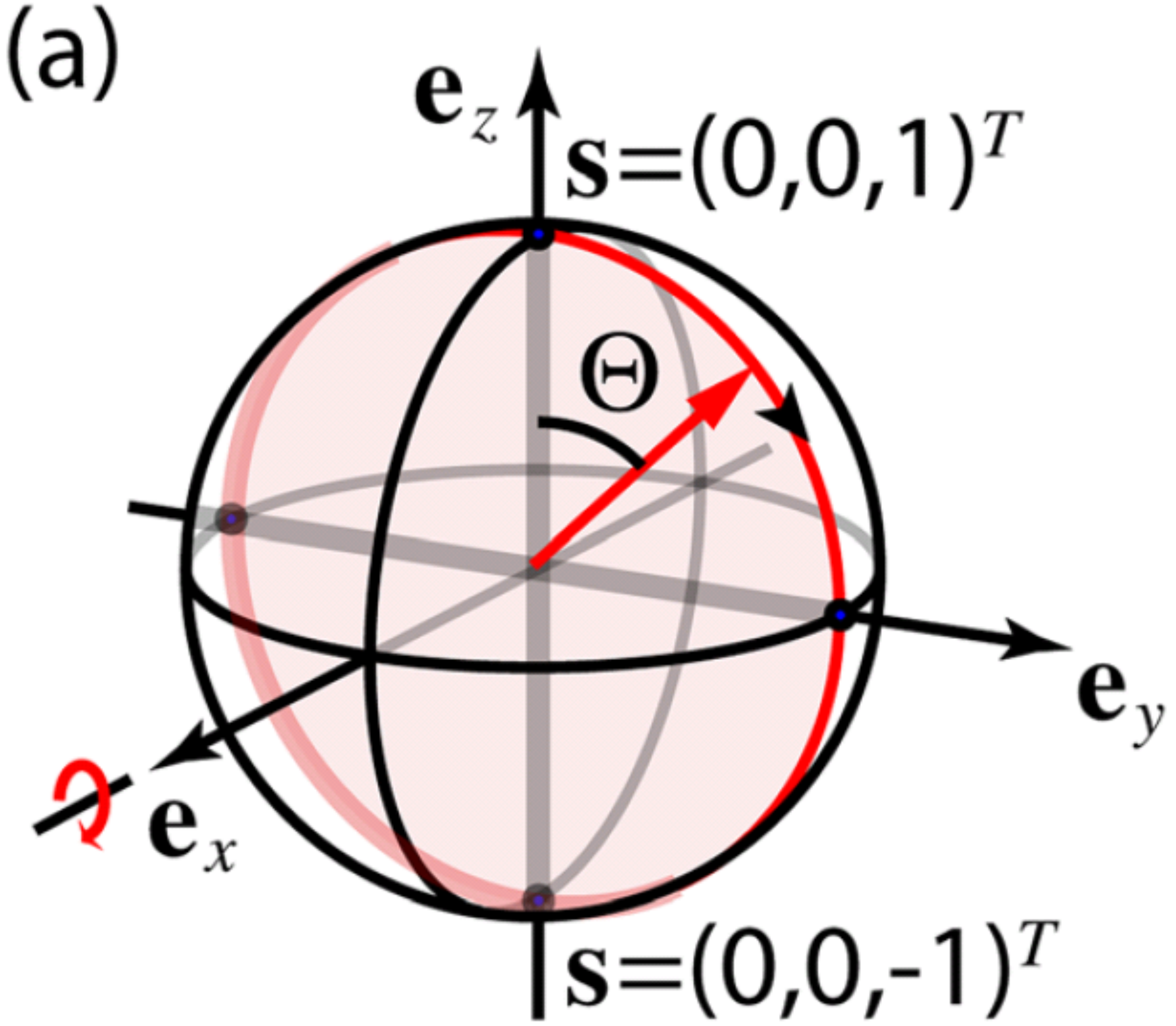
$$\mathcal{H} = \hbar\Omega S_x + \hbar\Delta S_z$$

detuning

tilted rotation axis $J = \begin{pmatrix} \Omega \\ 0 \\ \Delta \end{pmatrix}$

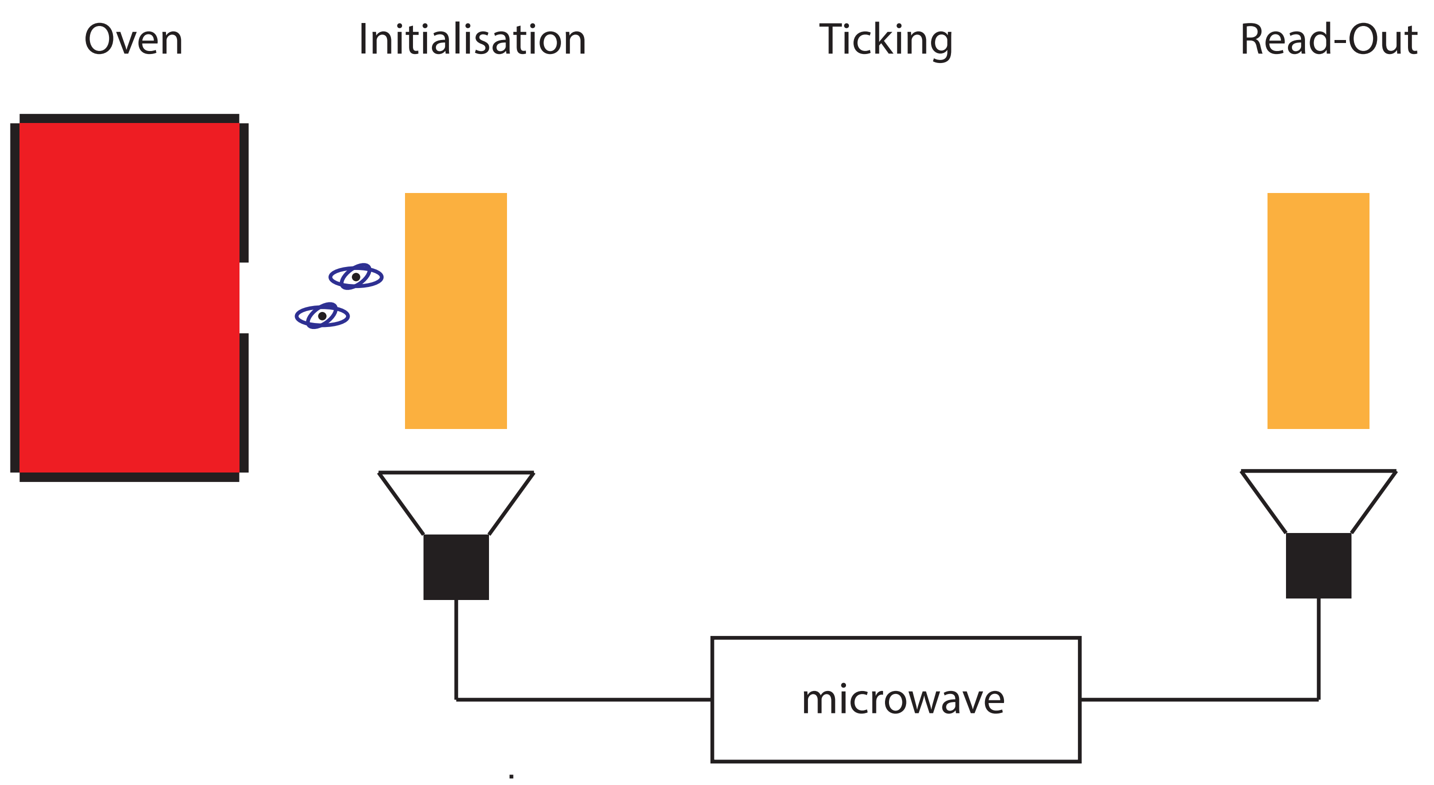
$$\Omega_{\text{eff}} = \sqrt{\Delta^2 + \Omega^2}$$

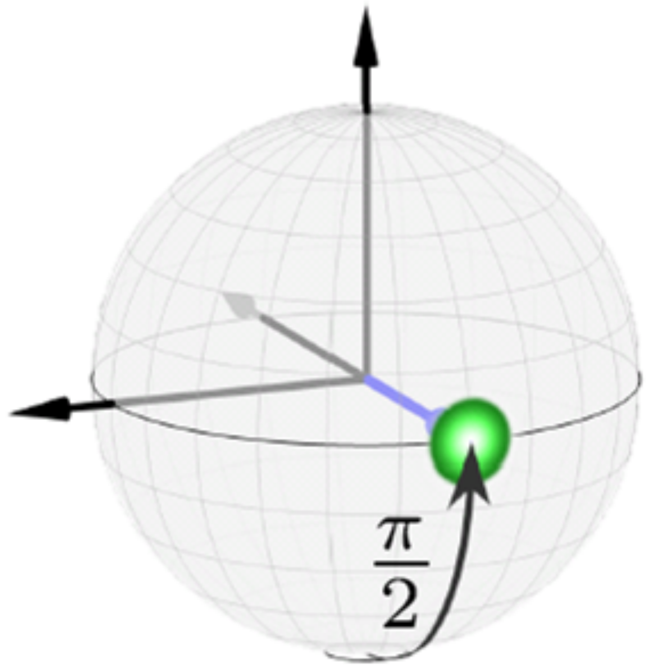


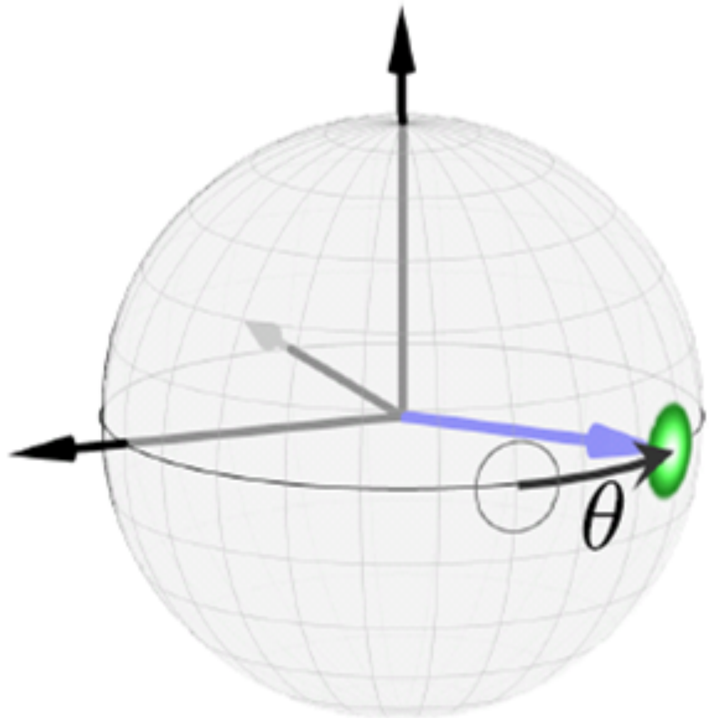


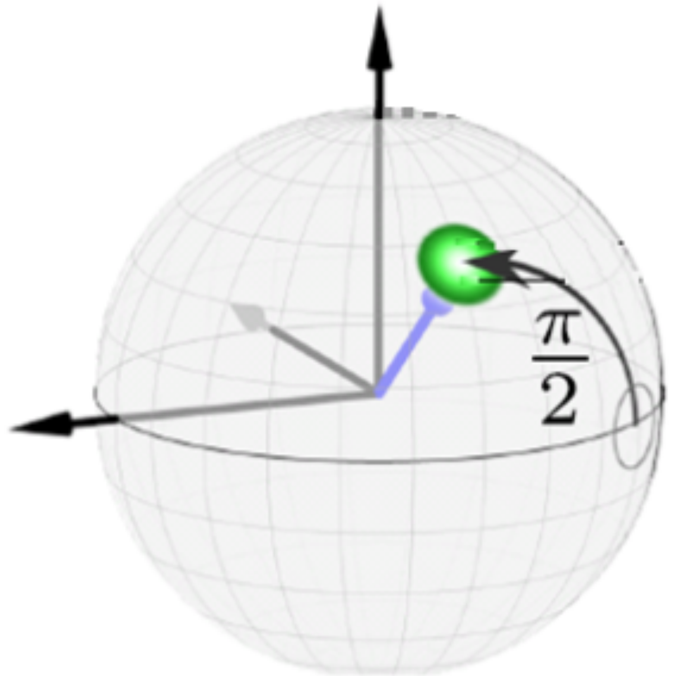
U_2

Back to our atomic clocks









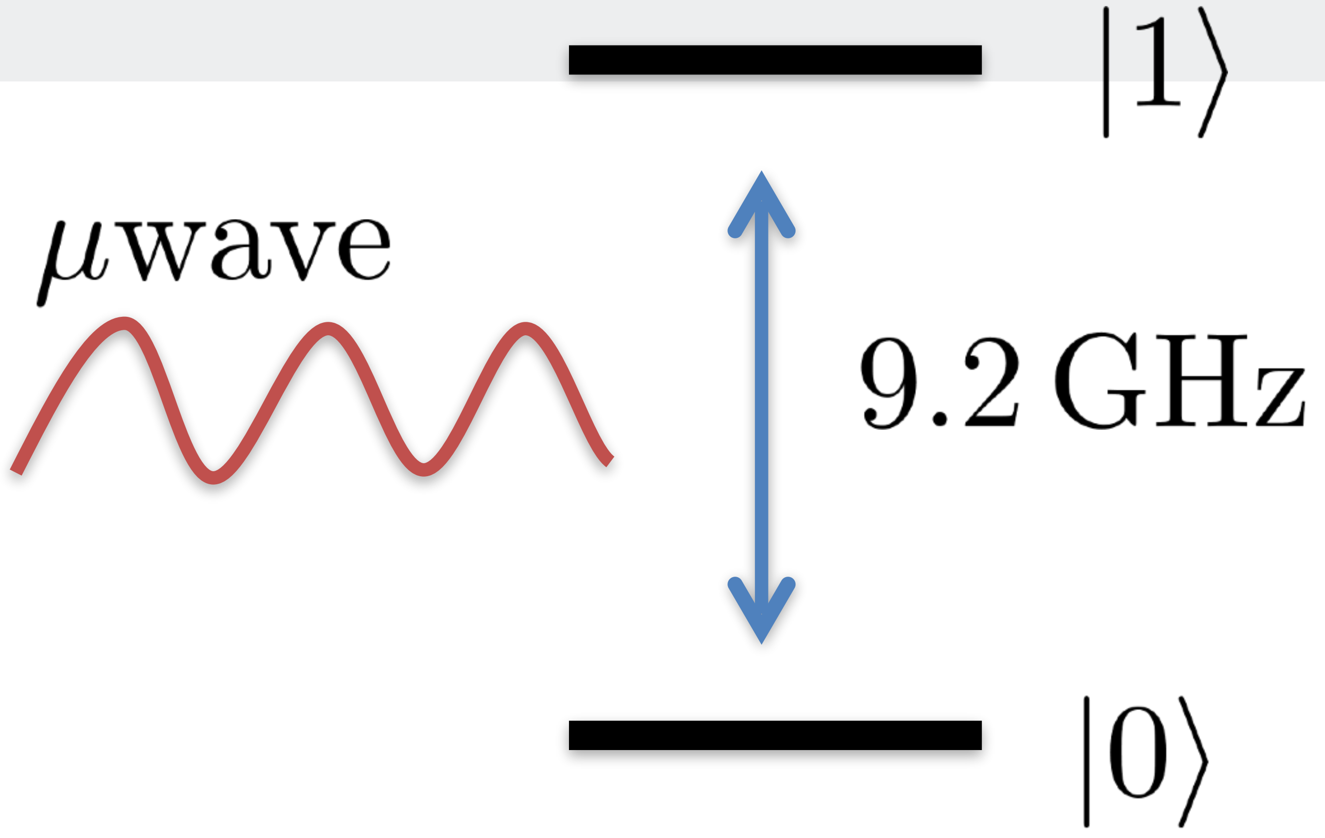
$X_{\pi/2}$

R_z

$X_{\pi/2}$

Application: Time standard with Cesium fountain clock

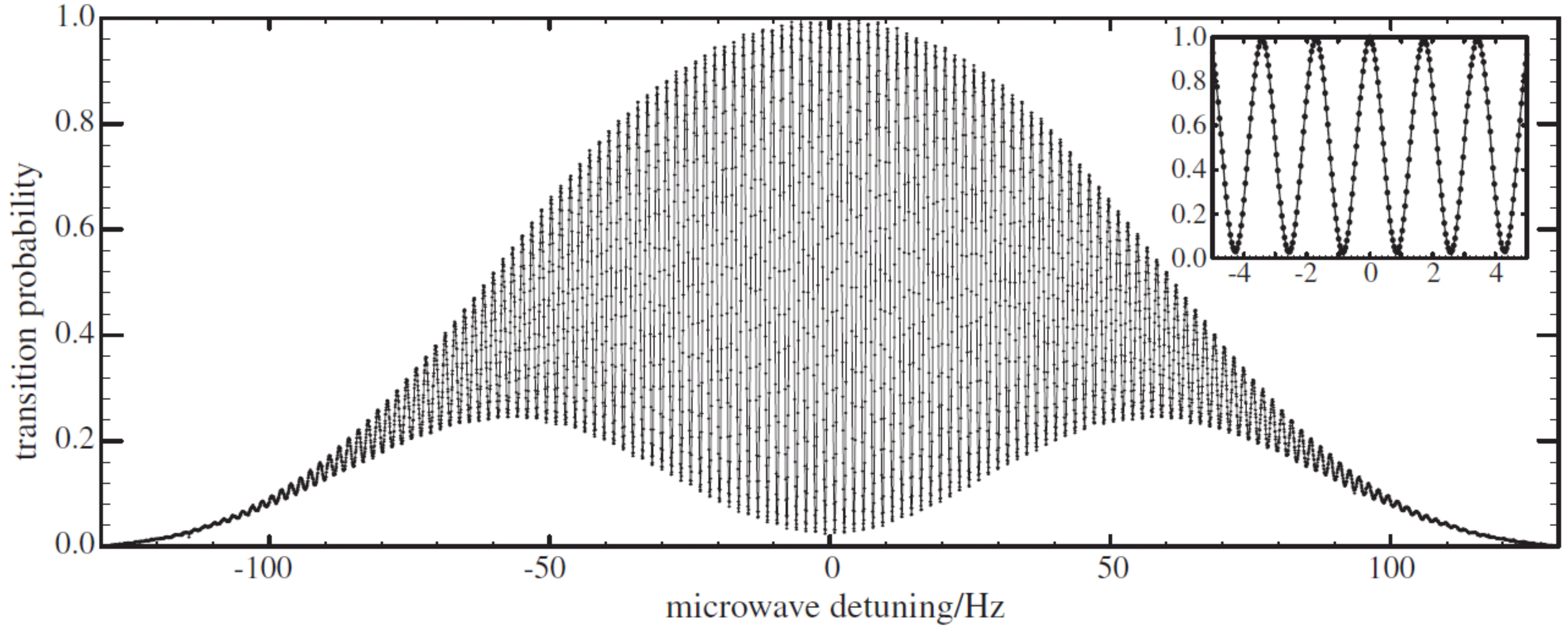
^{133}Cs: Hyperfine splitting 9.2 GHz

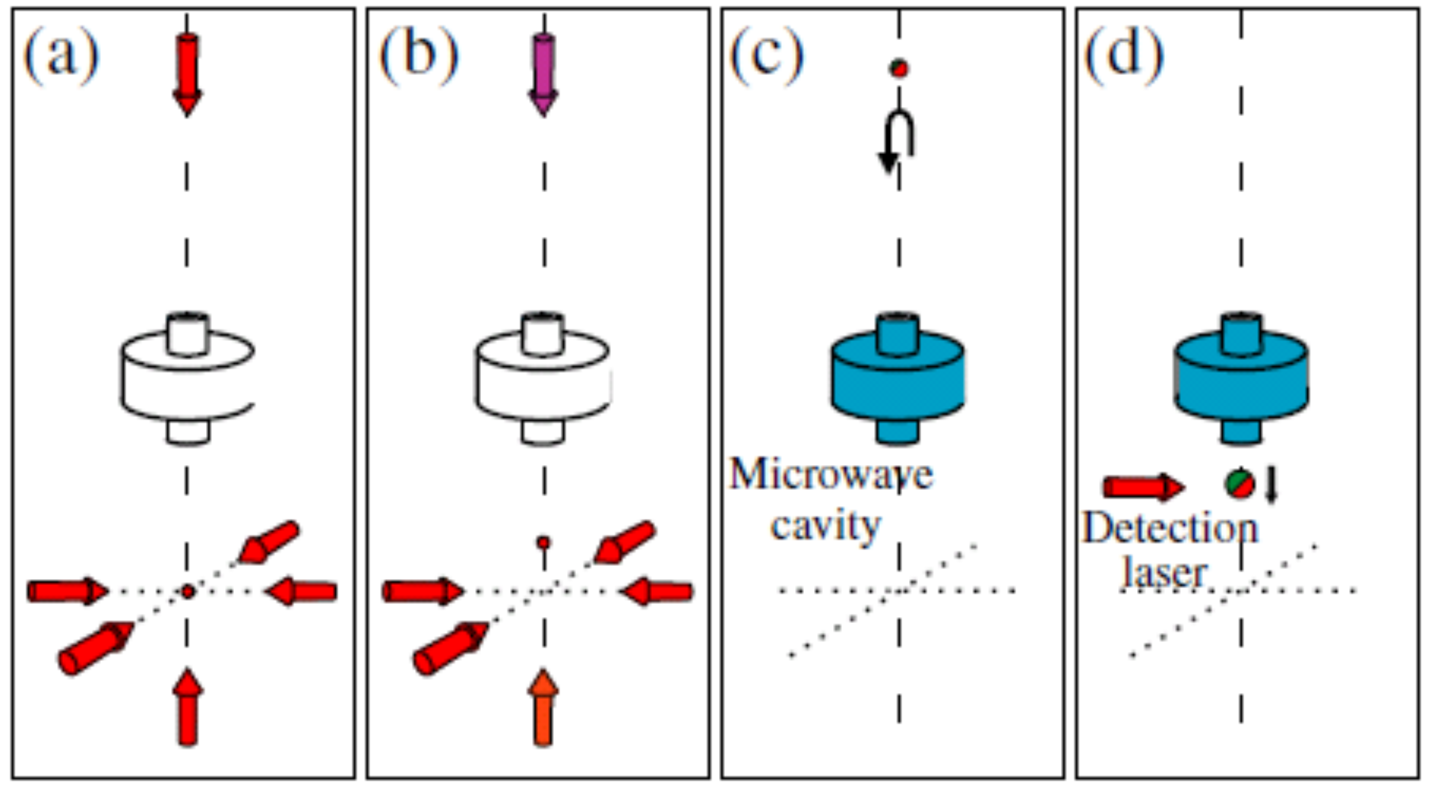


$$\text{envelope} \times \cos \Delta E t/\hbar$$

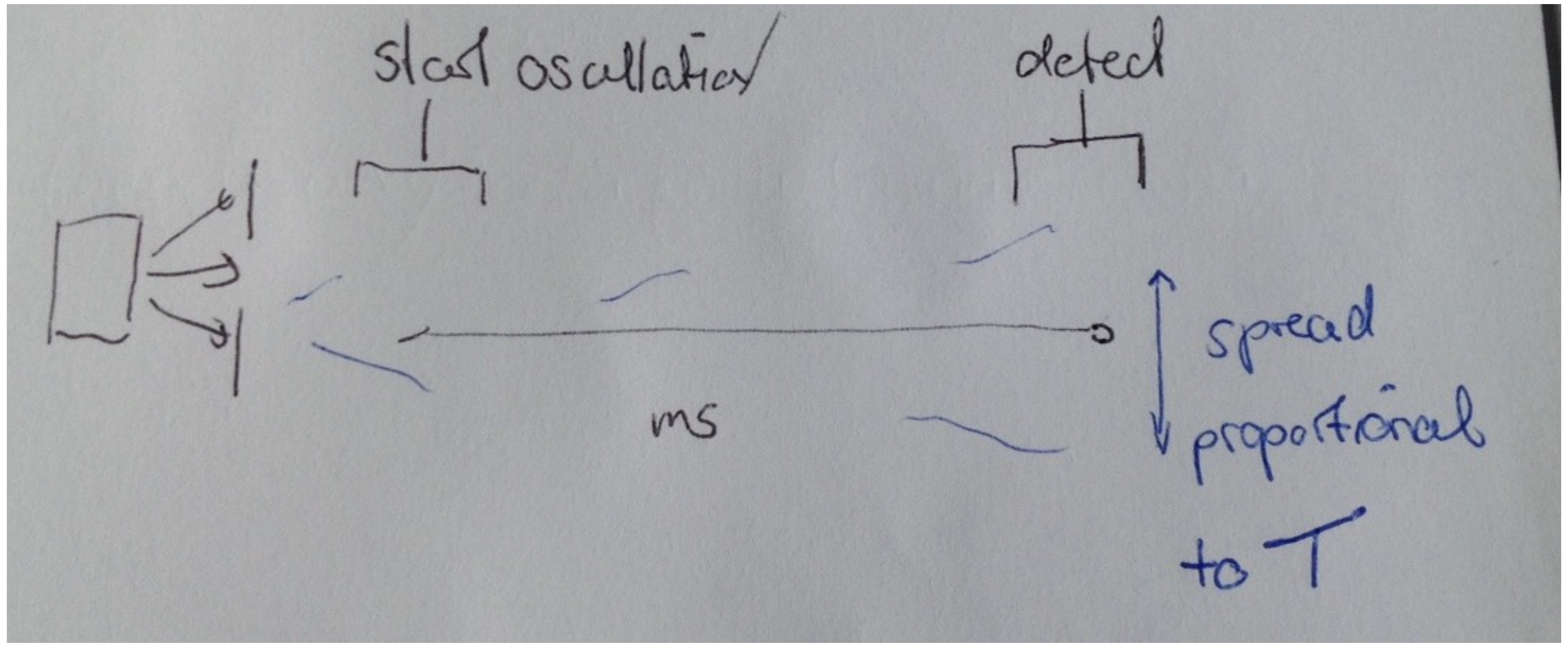
$$\Delta\omega \times \Delta t \geq 1$$

$$\rightarrow \text{precision: } \frac{\Delta\omega}{\omega} \times \frac{1}{\sqrt{N}} \approx 10^{-13}$$



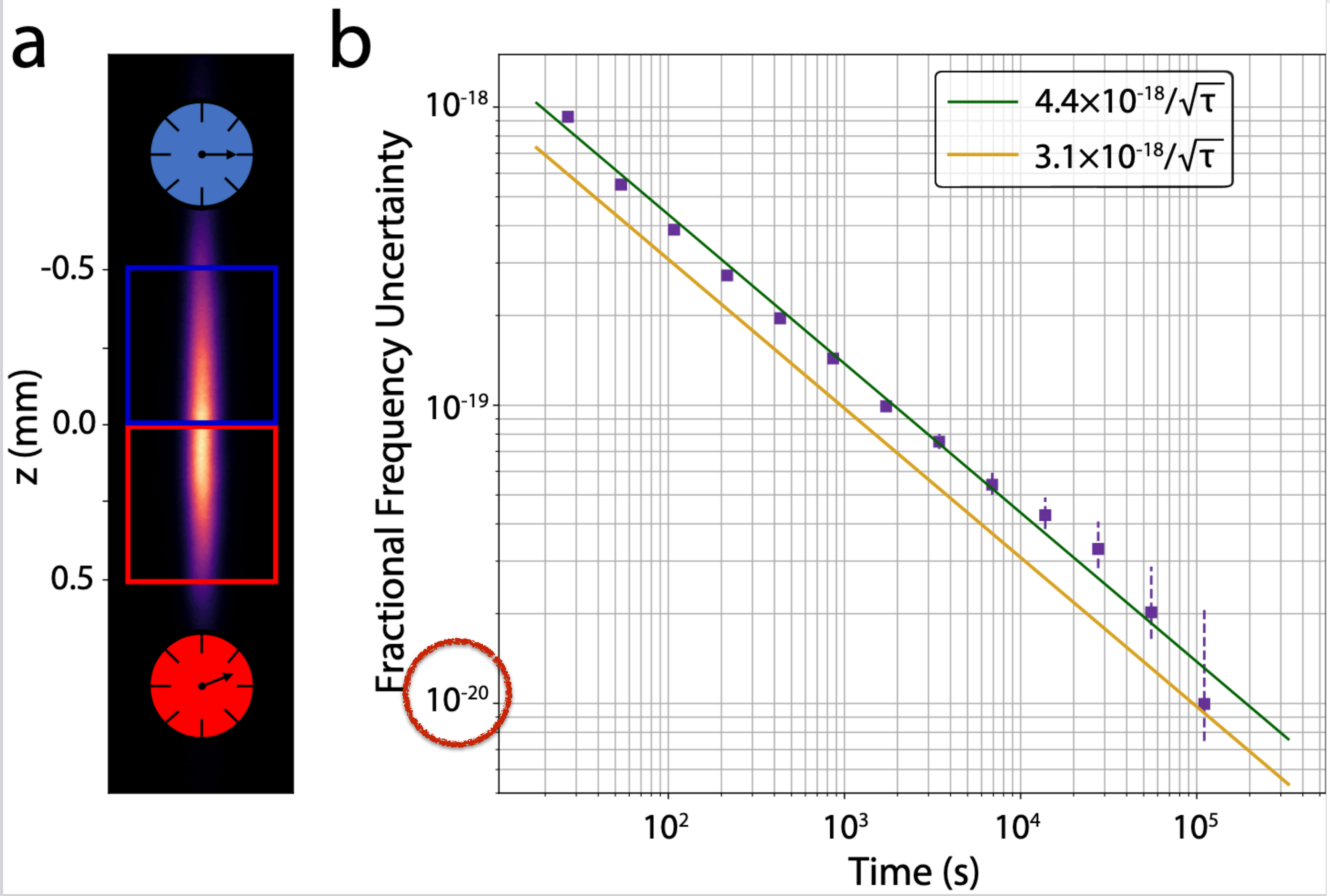


Ramsey limitations



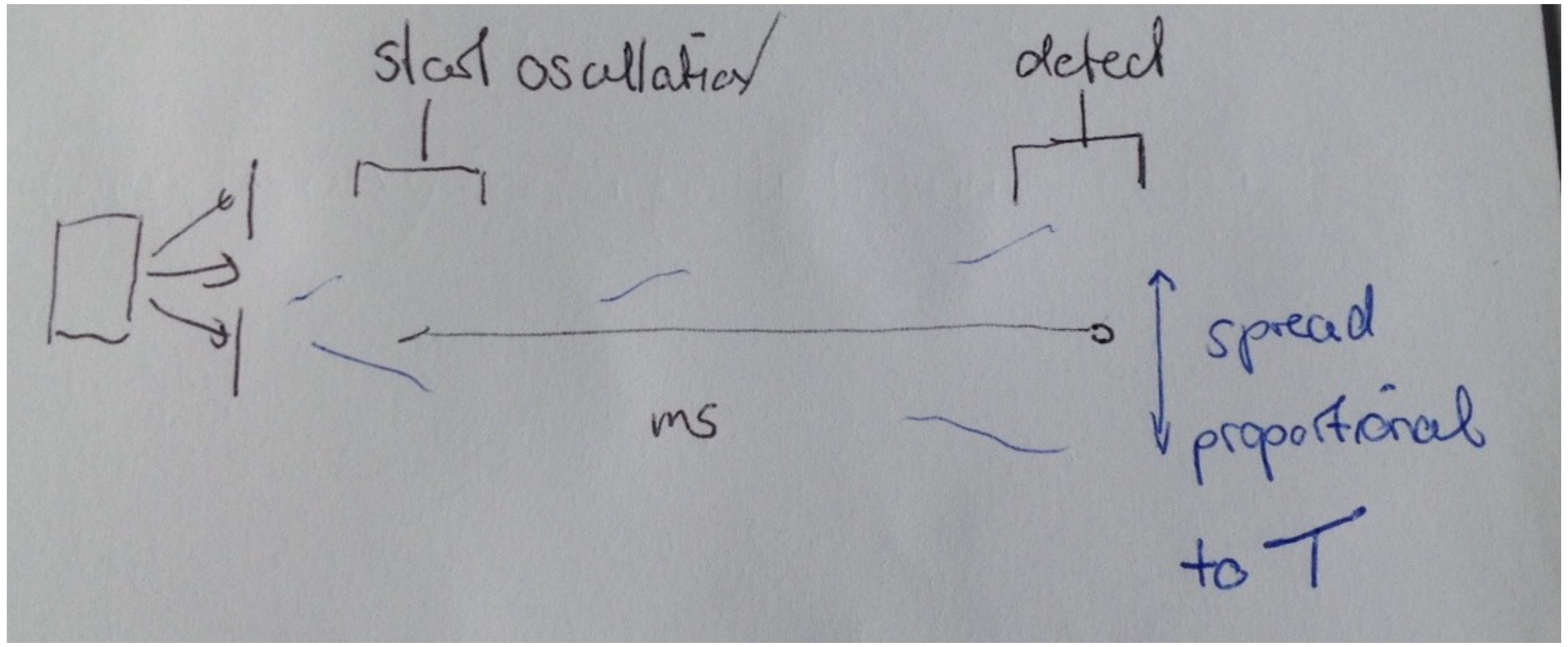
Detection better if atoms are slower

Measuring the red-shift on the millimeter scale



Bothwell et al. arXiv:2109.12238 (2021)

Ramsey limitations



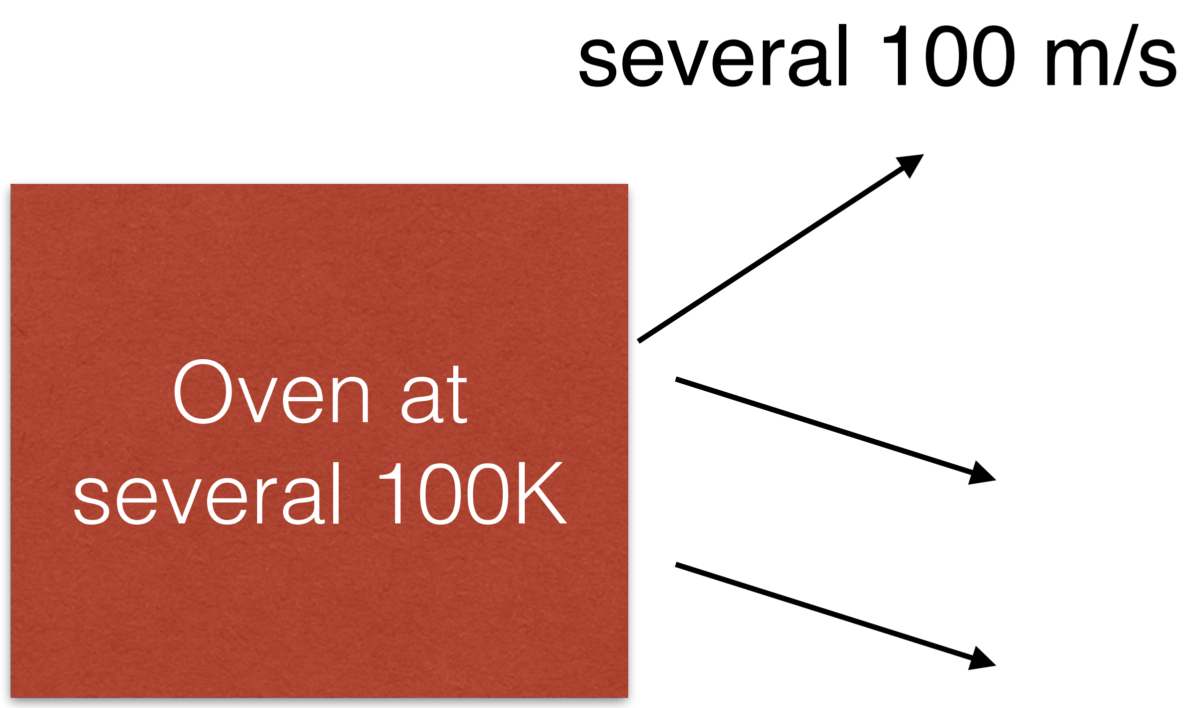
Detection better if atoms are slower

- We know how to perform qubit operation.
- How can we cool these atoms ?
- How can we trap them individually ?

1. Atomic clocks — Qubits in cold atoms
2. Optical tweezers — Trapped qubits in atoms
3. Rydberg atoms — Large scale entanglement
4. Moving particles — Bosons vs Fermions and the link to chemistry
5. Lattice gauge theories — Working on a really physics hard problem

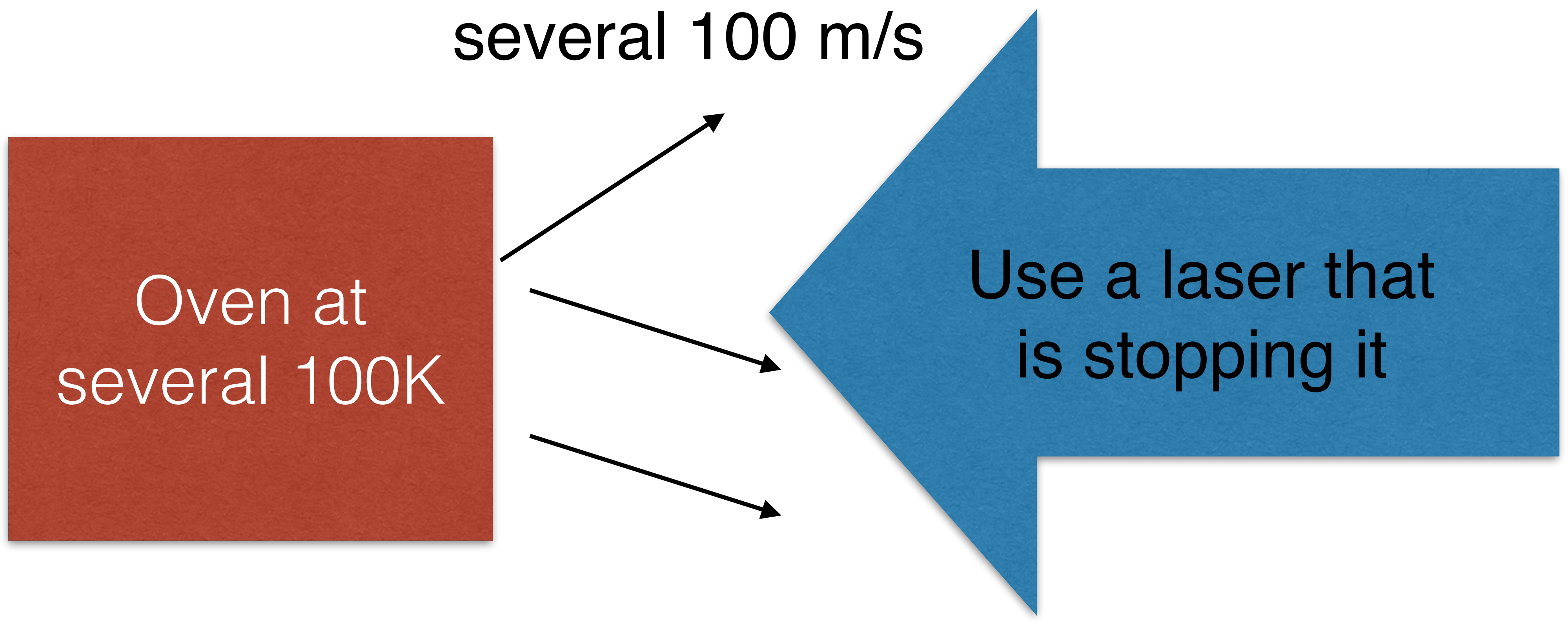
The idea of laser cooling

How to stop the atoms?



The idea of laser cooling

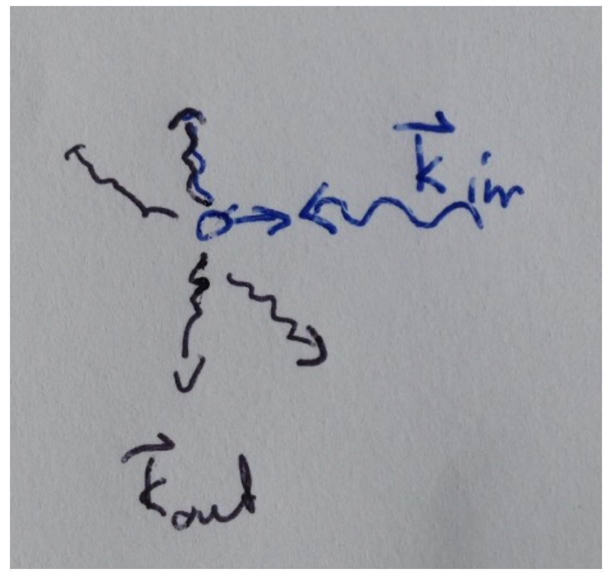
How to stop the atoms?



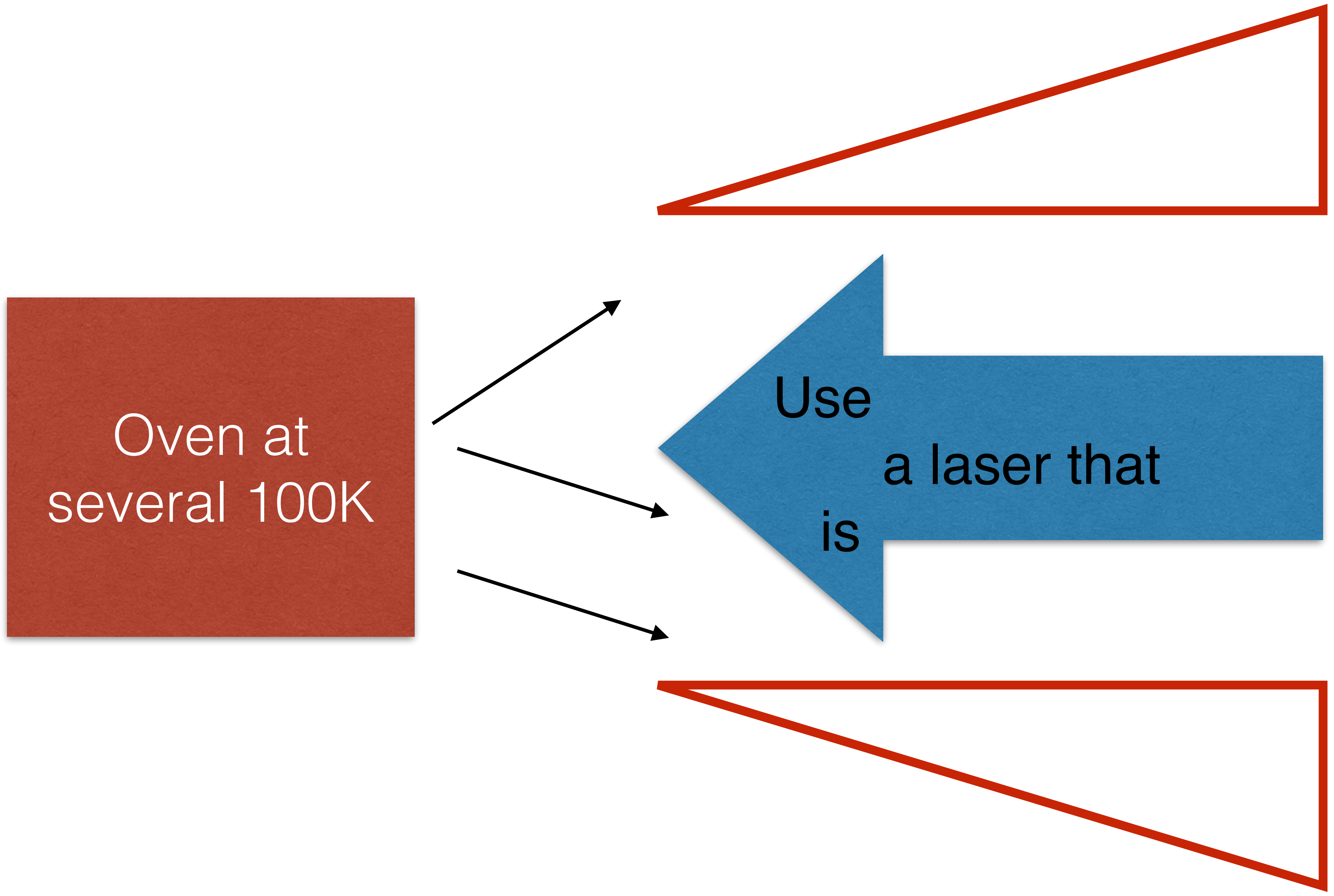
Some really
cold atoms

Idea of Laser cooling by Wineland, Dehmelt, Hänsch and Schawlow (1975)

Microscopic idea of radiation pressure



The Zeeman slower



a few m/s

VOLUME 48, NUMBER 9 PHYSICAL REVIEW LETTERS 1 MARCH 1982

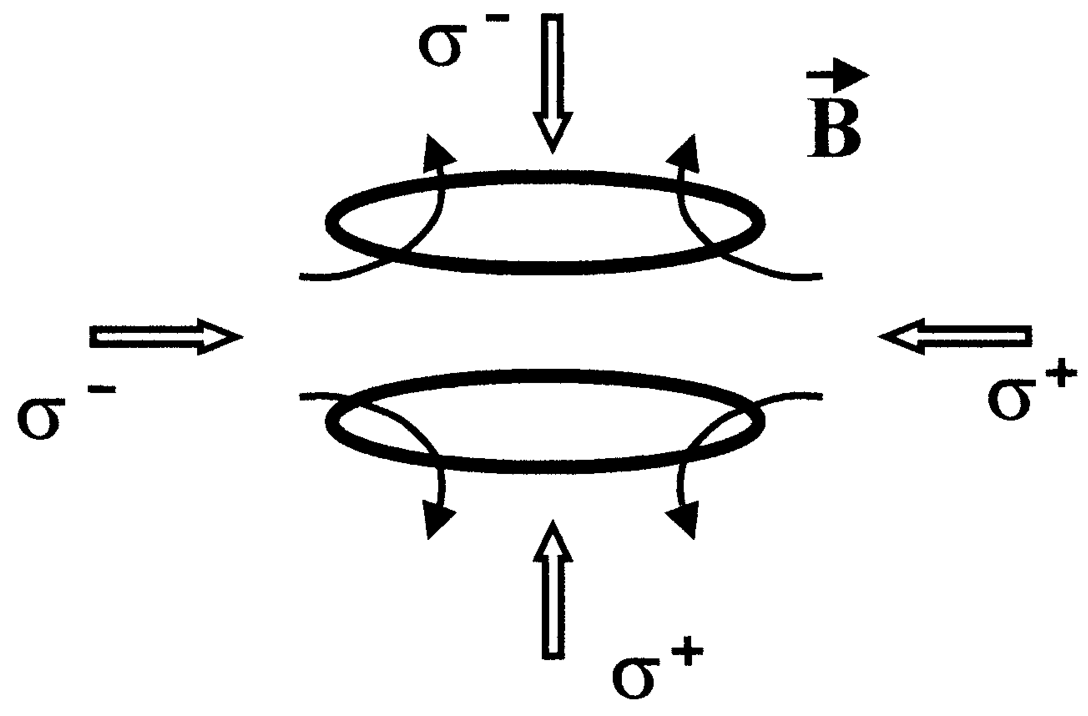
Laser Deceleration of an Atomic Beam

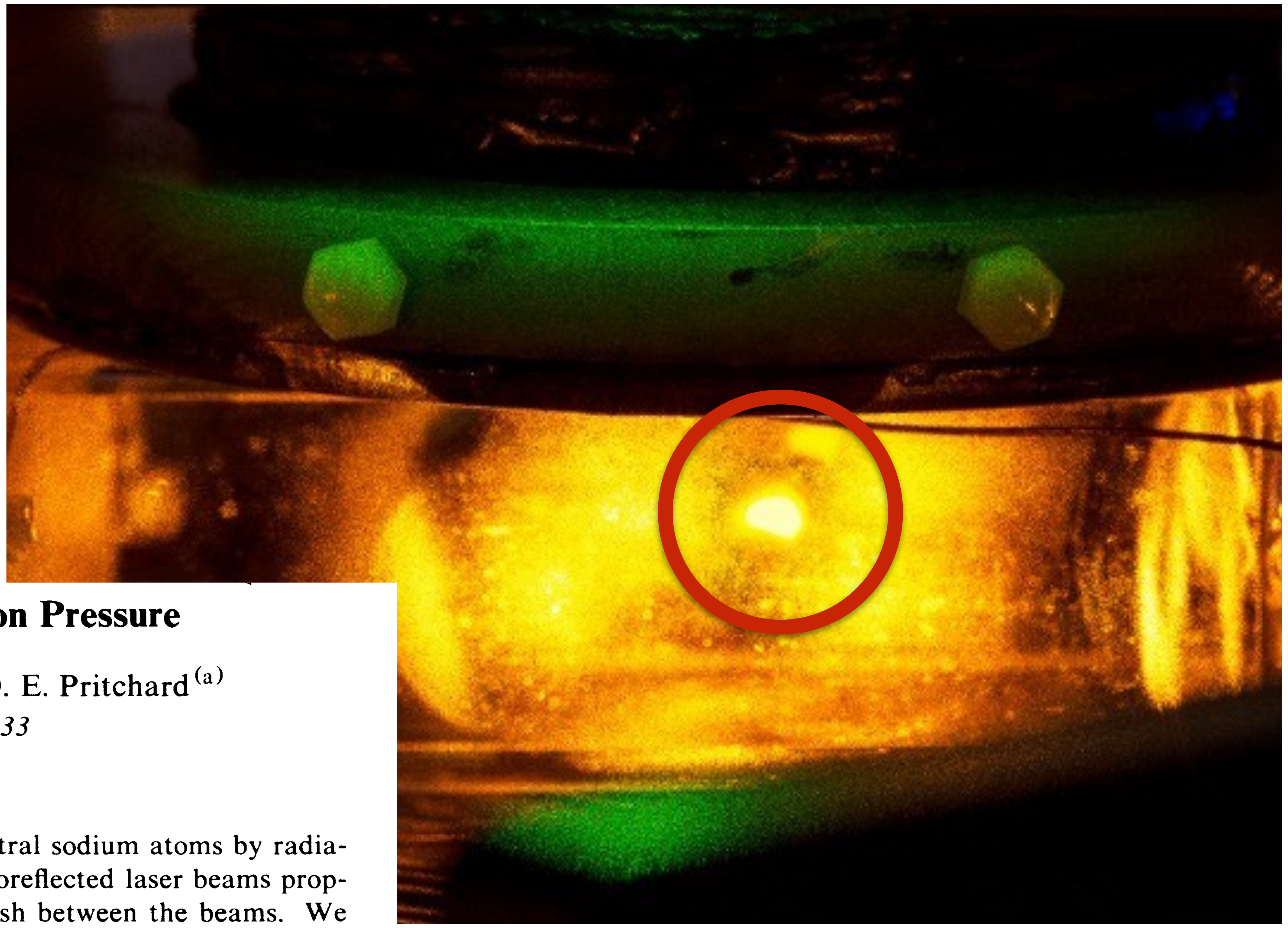
William D. Phillips and Harold Metcalf[a]
Electrical Measurements and Standards Division Center for Absolute Physical Quantities, National Bureau of Standards, Washington, D. C. 20234
(Received 23 December 1981)

Deceleration and velocity bunching of Na atoms in an atomic beam have been observed. The deceleration, caused by absorption of counterpropagating resonant laser light, amounts to 40% of the initial thermal velocity, corresponding to about 15 000 absorptions. Atoms were kept in resonance with the laser by using a spatially varying magnetic field to provide a changing Zeeman shift to compensate for the changing Doppler shift as the atoms decelerated.

The MOT

static magnetic fields + radiation pressure



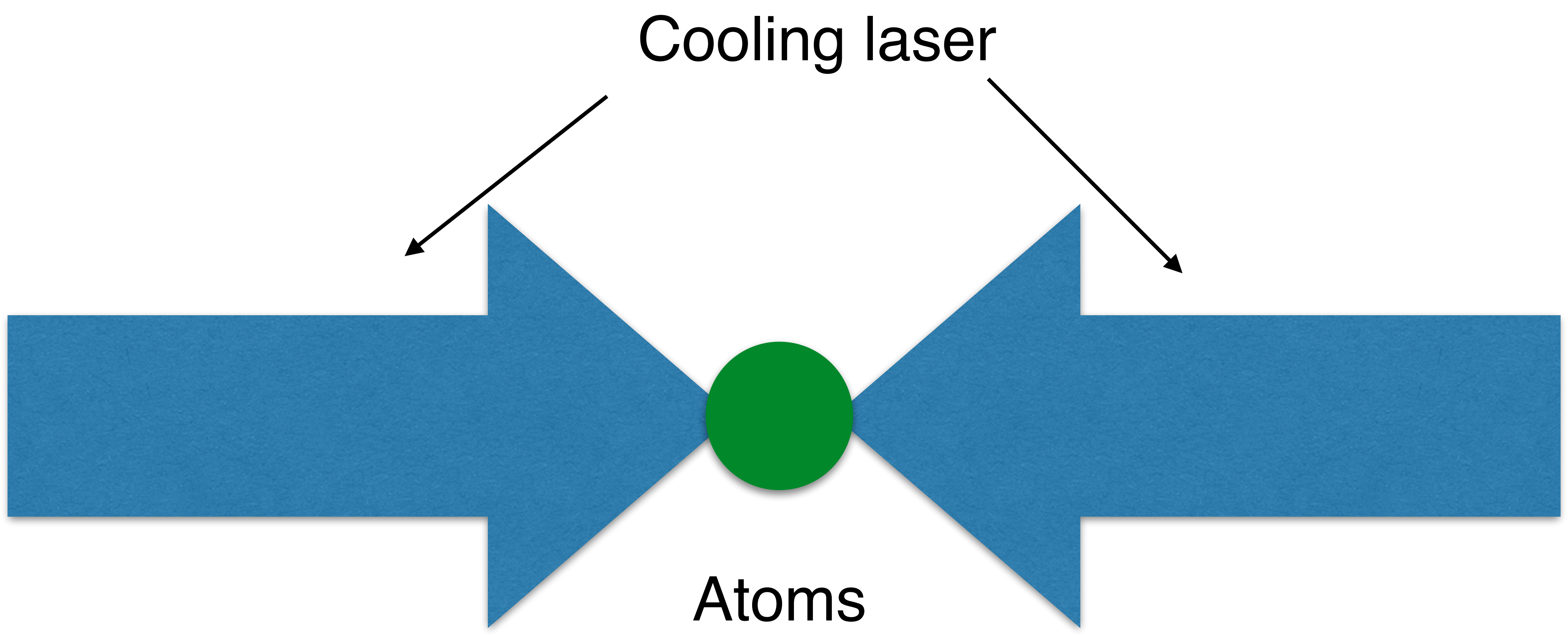


Trapping of Neutral Sodium Atoms with Radiation Pressure

E. L. Raab,[a] M. Prentiss, Alex Cable, Steven Chu,[b] and D. E. Pritchard[a]
AT&T Bell Laboratories, Holmdel, New Jersey 07733
(Received 16 July 1987)

We report the confinement and cooling of an optically dense cloud of neutral sodium atoms by radiation pressure. The trapping and damping forces were provided by three retroreflected laser beams propagating along orthogonal axes, with a weak magnetic field used to distinguish between the beams. We have trapped as many as 10^7 atoms for 2 min at densities exceeding 10^{11} atoms cm^{-3}. The trap was $\simeq 0.4$ K deep and the atoms, once trapped, were cooled to less than a millikelvin and compacted into a region less than 0.5 mm in diameter.

Doppler Cooling/Optical Molasses



atoms undergo diffusive motion and feel ‚friction’ from collisions with laser

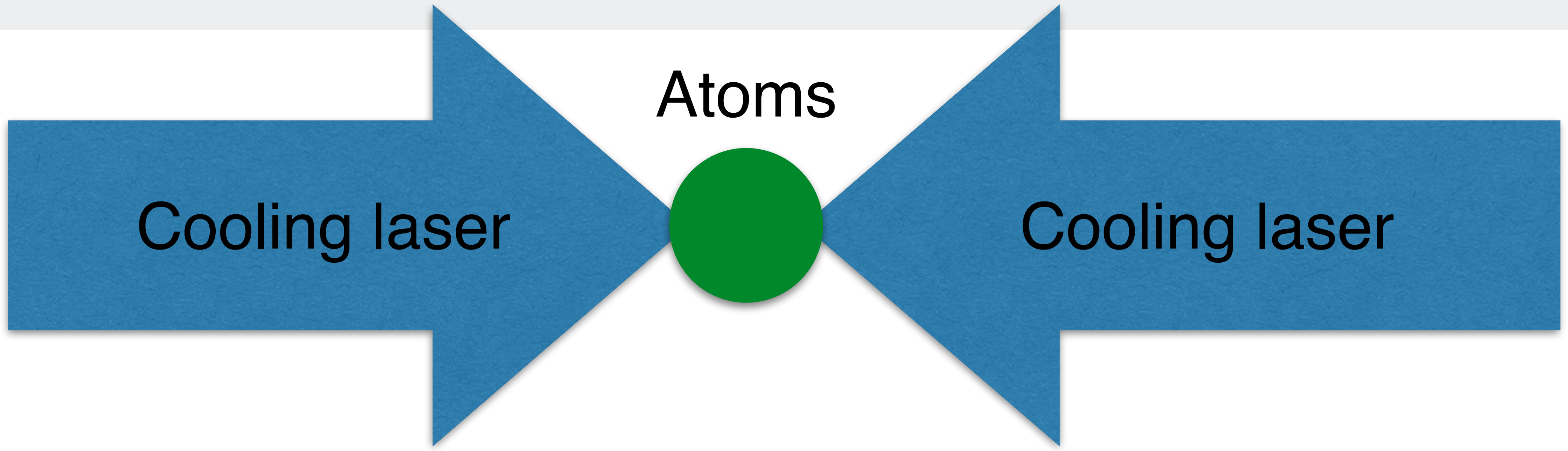
VOLUME 55, NUMBER 1 PHYSICAL REVIEW LETTERS 1 JULY 1985

Three-Dimensional Viscous Confinement and Cooling of Atoms by Resonance Radiation Pressure

Steven Chu, L. Hollberg, J. E. Bjorkholm, Alex Cable, and A. Ashkin
AT&T Bell Laboratories, Holmdel, New Jersey 07733
(Received 25 April 1985)

We report the viscous confinement and cooling of neutral sodium atoms in three dimensions via the radiation pressure of counterpropagating laser beams. These atoms have a density of about $\sim 10^6$ cm^{-3} and a temperature of ~ 240 μK corresponding to a rms velocity of ~ 60 cm/sec. This temperature is approximately the quantum limit for this atomic transition. The decay time for half the atoms to escape a ~ 0.2-cm^3 confinement volume is ~ 0.1 sec.

Doppler Cooling/Optical Molasses



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Doppler limit (lowest ‚possible' temperature) $T_D \approx 240 \mu K$

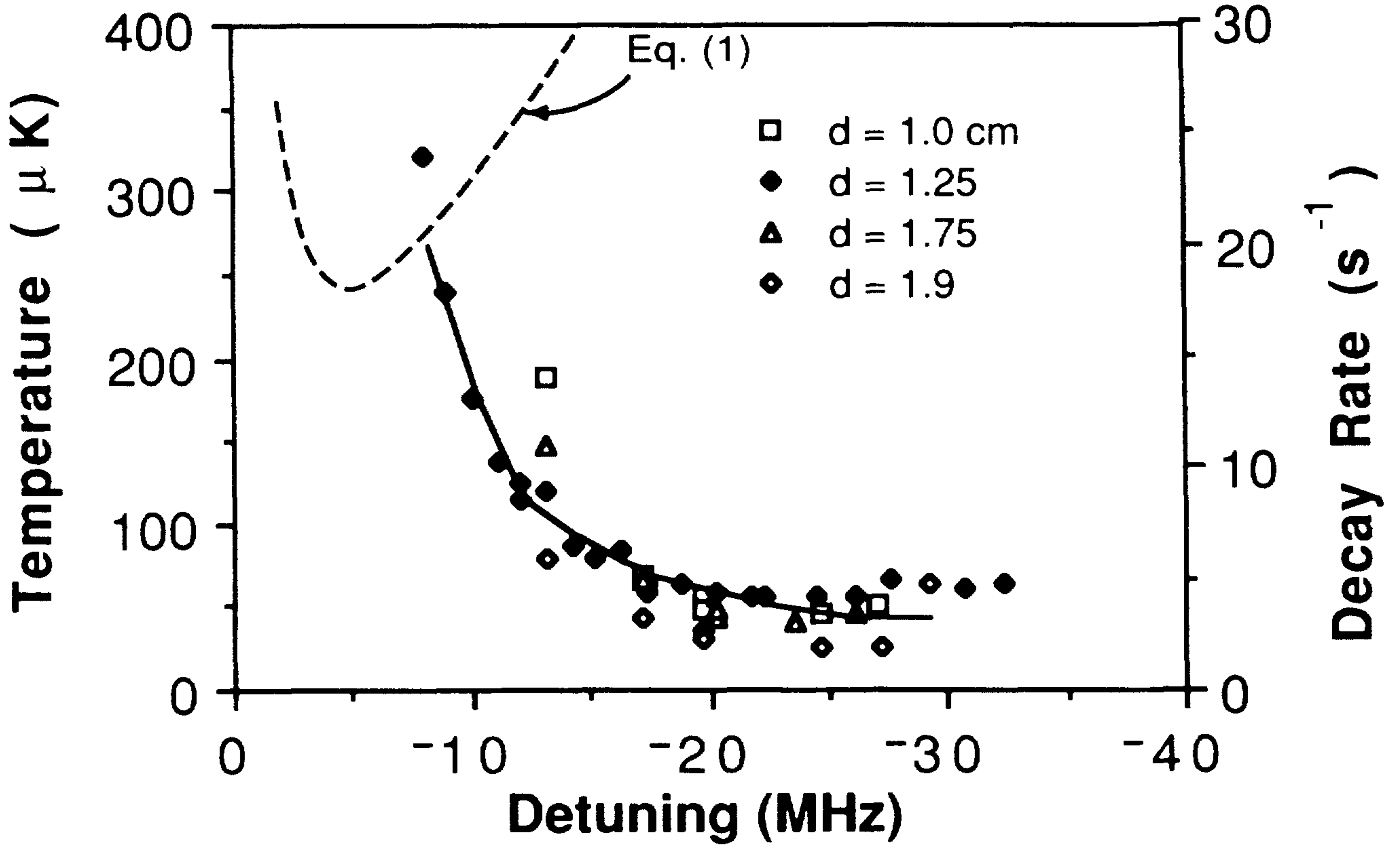
observed: $T \approx 240^{+200}_{-60} \mu K$

The miracle of subdoppler cooling

Comment by Steve Chu:

sure the velocity distribution. Our first measurements showed a temperature of 185 μK, slightly lower than the minimum temperature allowed by the theory of Doppler cooling. We then made the cardinal mistake of experimental physics: instead of listening to Nature, we were overly influenced by theoretical expectations. By including a fudge factor to account for the way atoms filled the molasses region, we were able to bring our measurement into accord with our expectations.

The result by the Phillips group:



Lett *et al.* PRL **61** 169 (1988)

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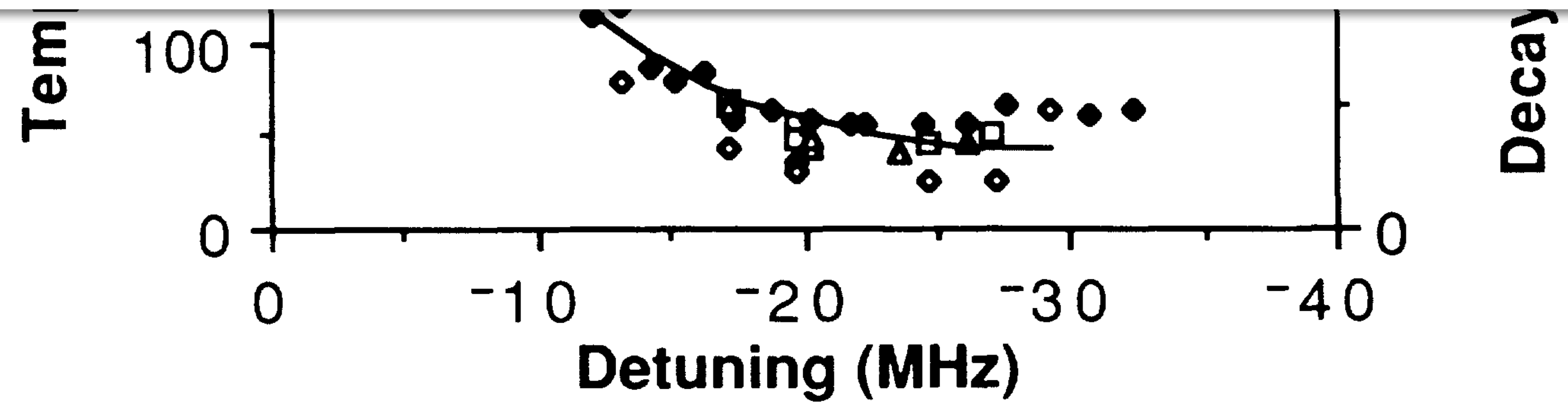
The result by the

Steven Chu   William Phillips   Claude Cohen-Tannoudji



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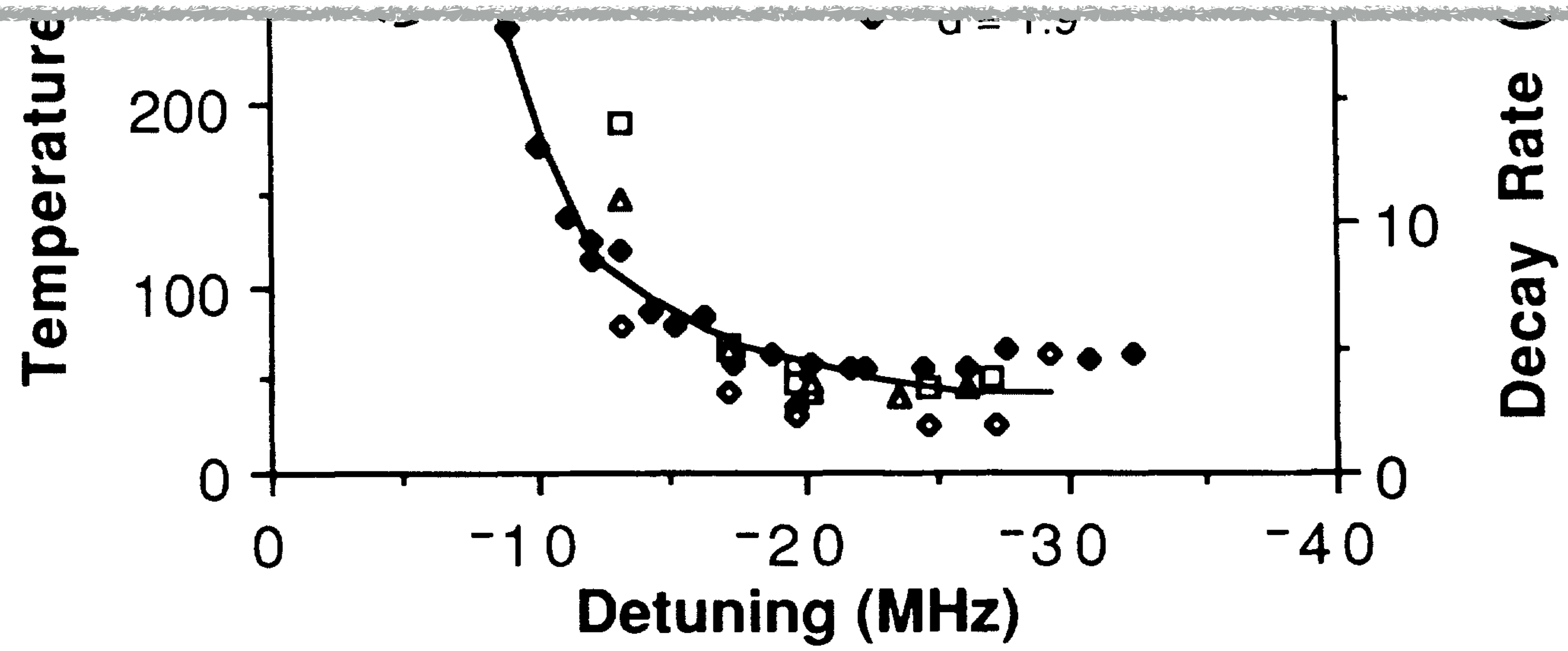
sure the velocity distribution. Our first measurements showed a temperature of 185 μK, slightly lower than the minimum temperature allowed by the theory of Doppler cooling. We then made the cardinal mistake of experimental physics: instead of listening to Nature, we were overly influenced by theoretical expectations. By including a fudge factor to account for the way atoms filled the molasses region, we were able to bring our measurement into accord with our expectations.

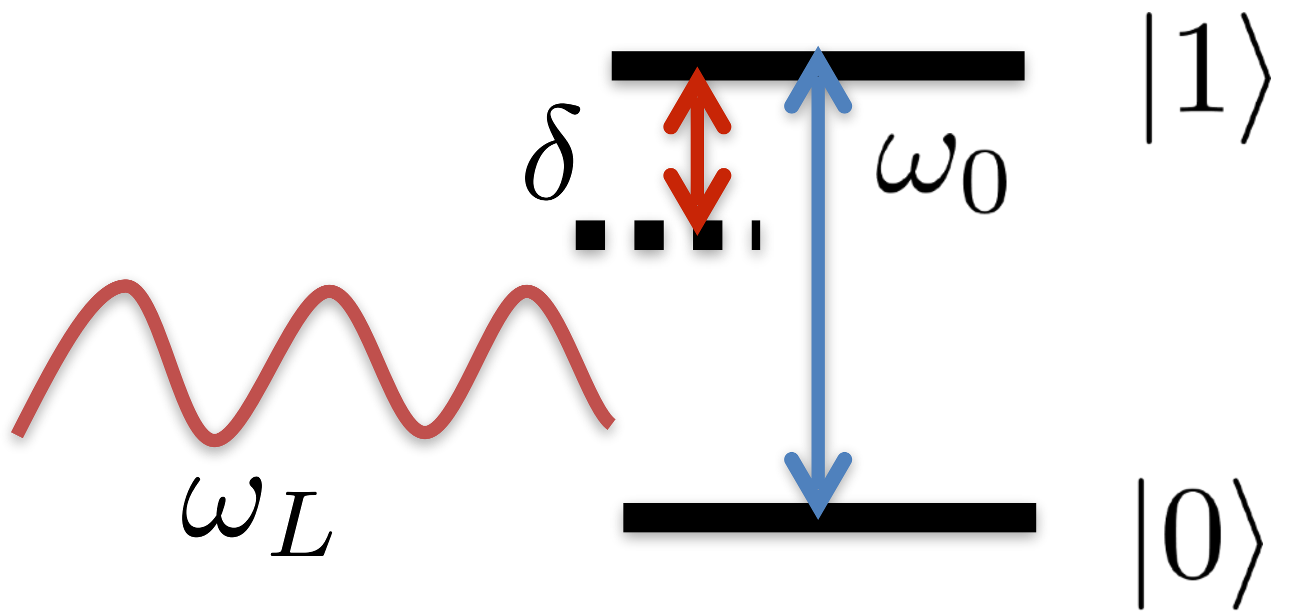
The result by

- We know how to perform qubit operation.
- Atoms are really cold.
- How can we trap them individually ?



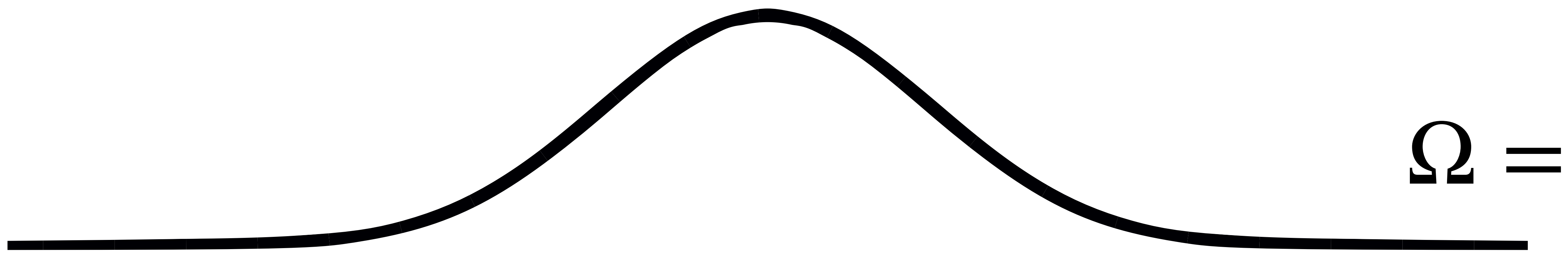
Lett *et al.* PRL **61** 169 (1988)

The dipole potential - making optical tweezers



$$\Omega = dE(r)$$

$$\delta = \omega_l - \omega_0$$

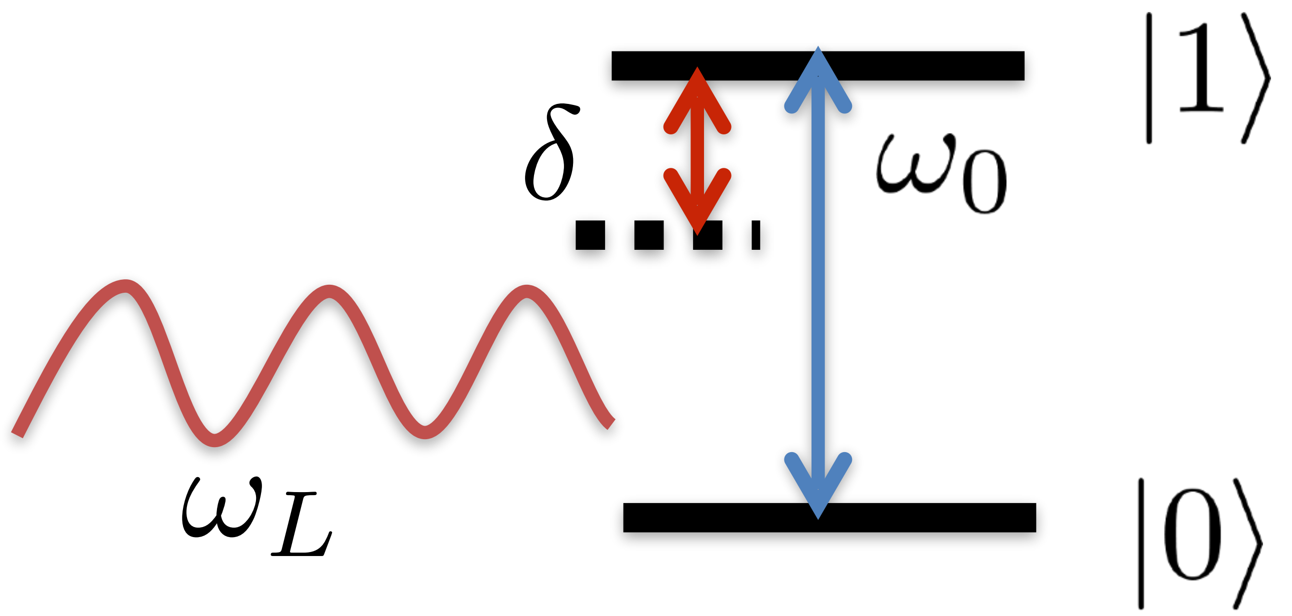


$$\Omega = dE(r)$$

$$E \propto \frac{\Omega^2(r)}{\delta} \propto \frac{I(r)}{\delta}$$

The dipole potential is directly proportional to the intensity !

The dipole potential - making optical tweezers



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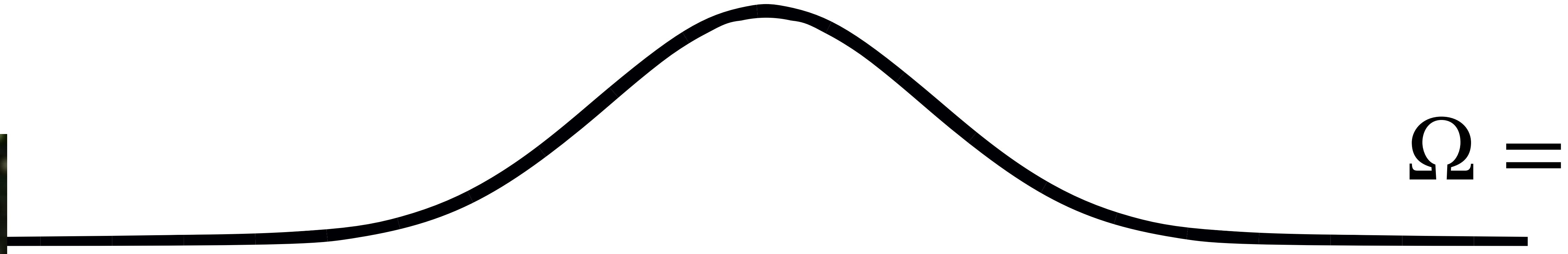
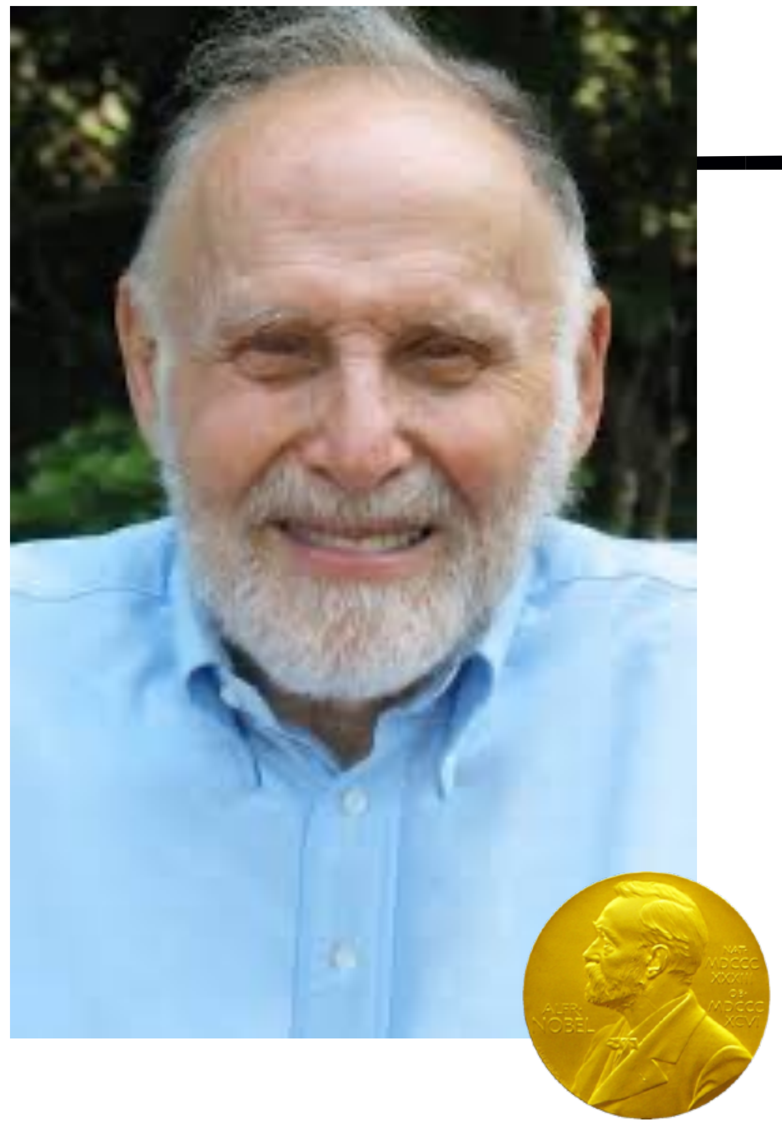


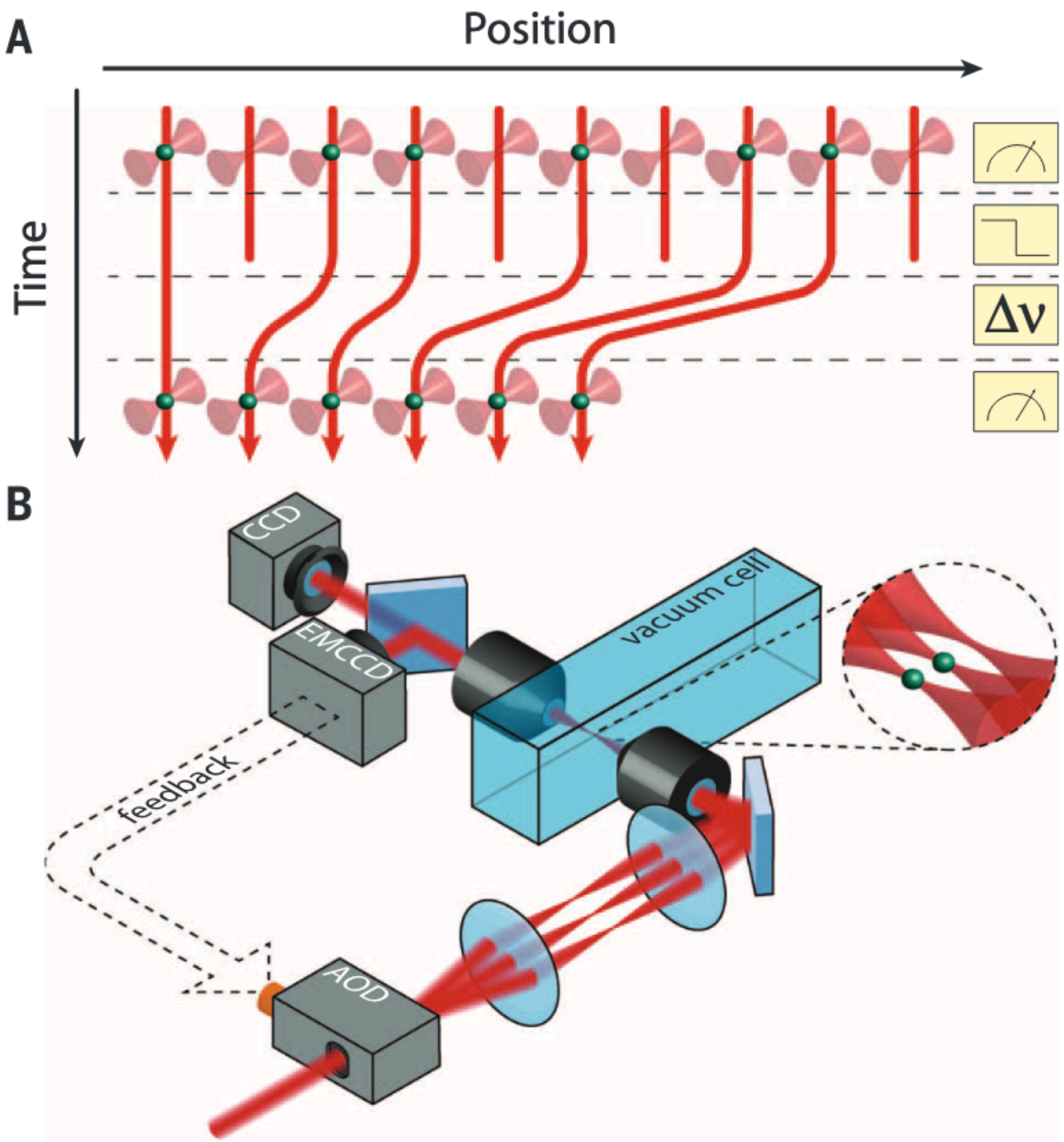
$$\Omega = dE(r)$$



$$E \propto \frac{\Omega^2(r)}{\delta} \propto \frac{I(r)}{\delta}$$

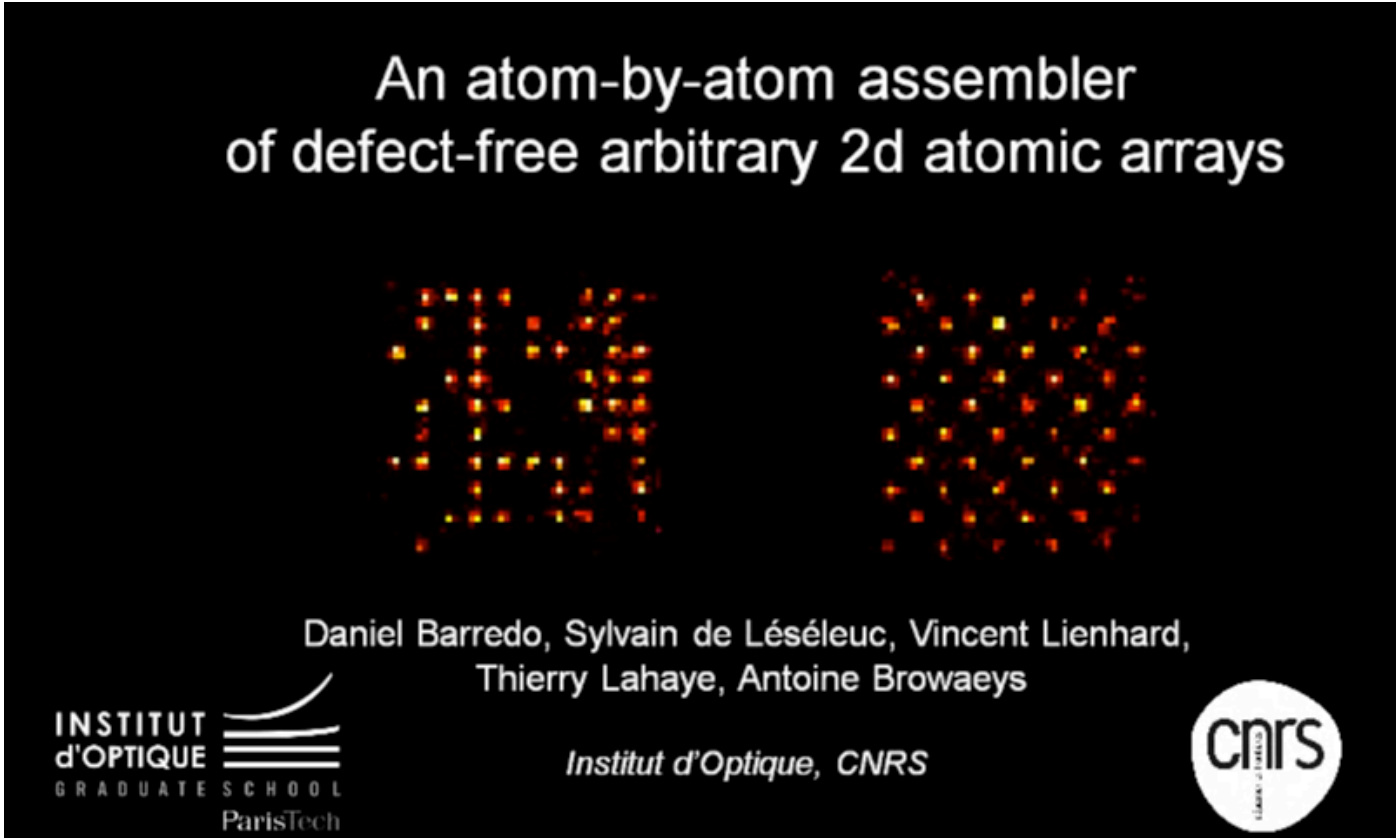
The dipole potential is directly proportional to the intensity !

Tweezers and atom sorting



M. Endres et al., Science 354, 1024 (2016).

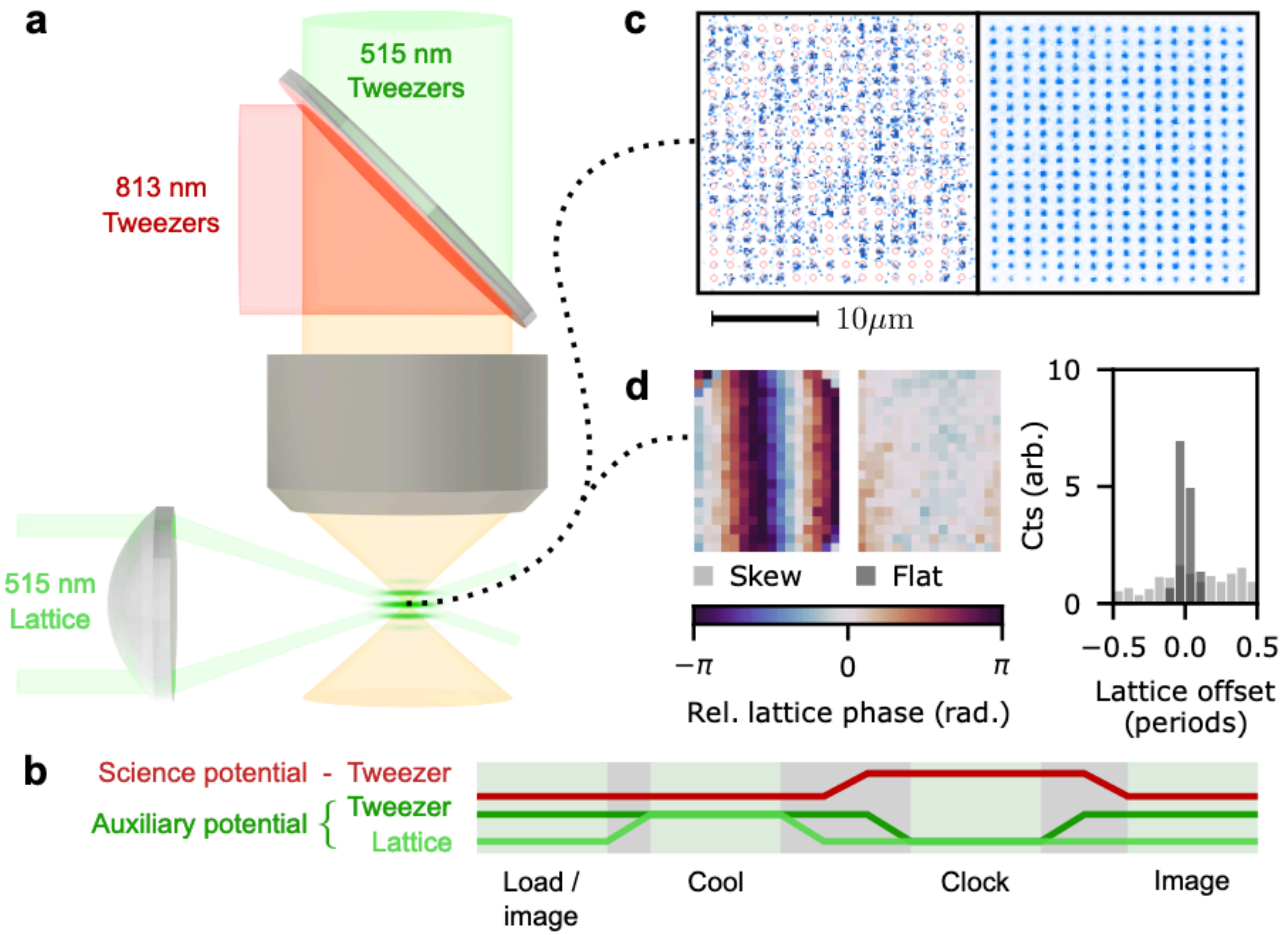
Tweezers and atom sorting

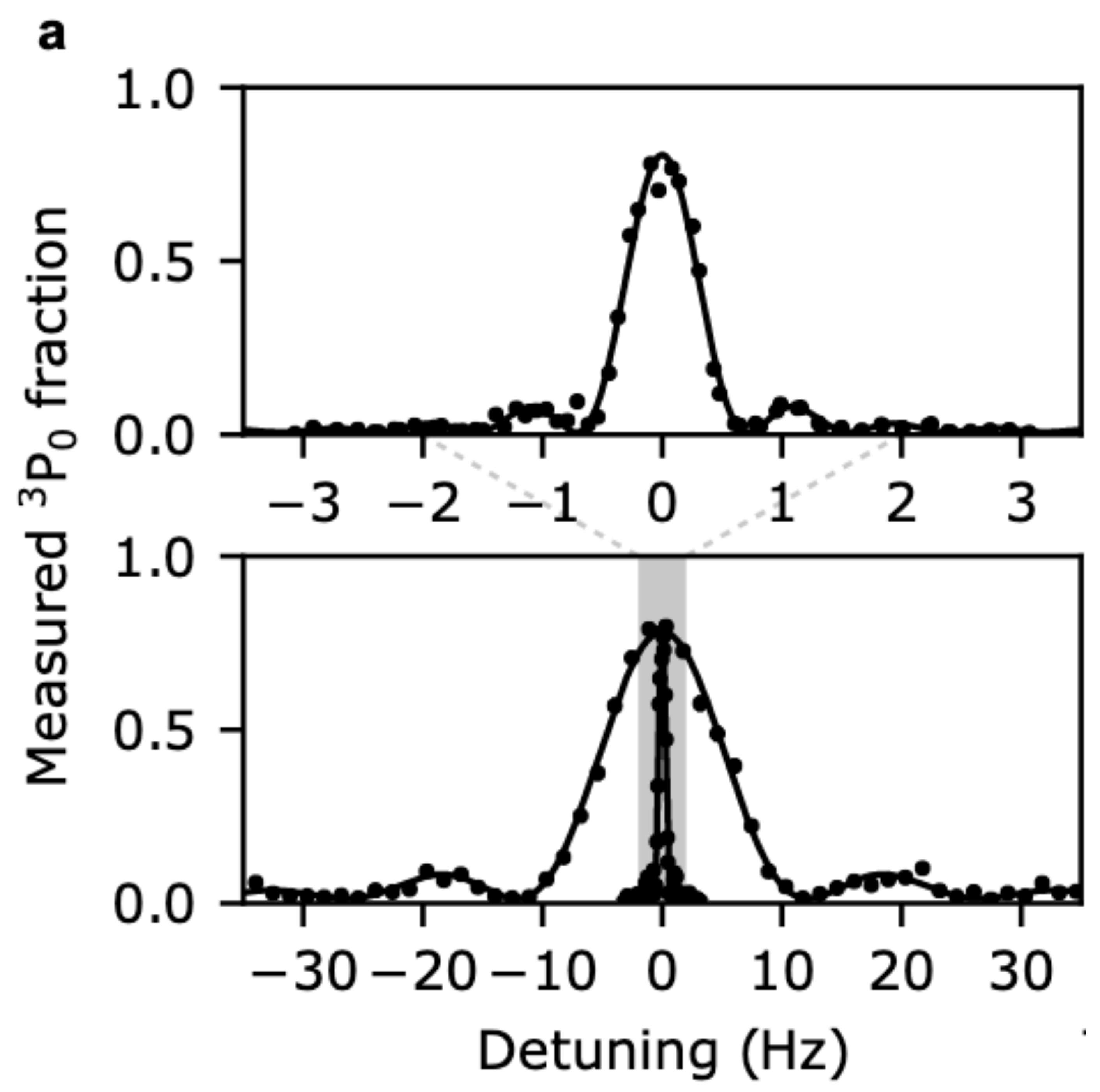


D. Barredo et al. Science 354, 1021 (2016).

Tweezer clocks

A. Young et al. arXiv:2004.06095 (2020)

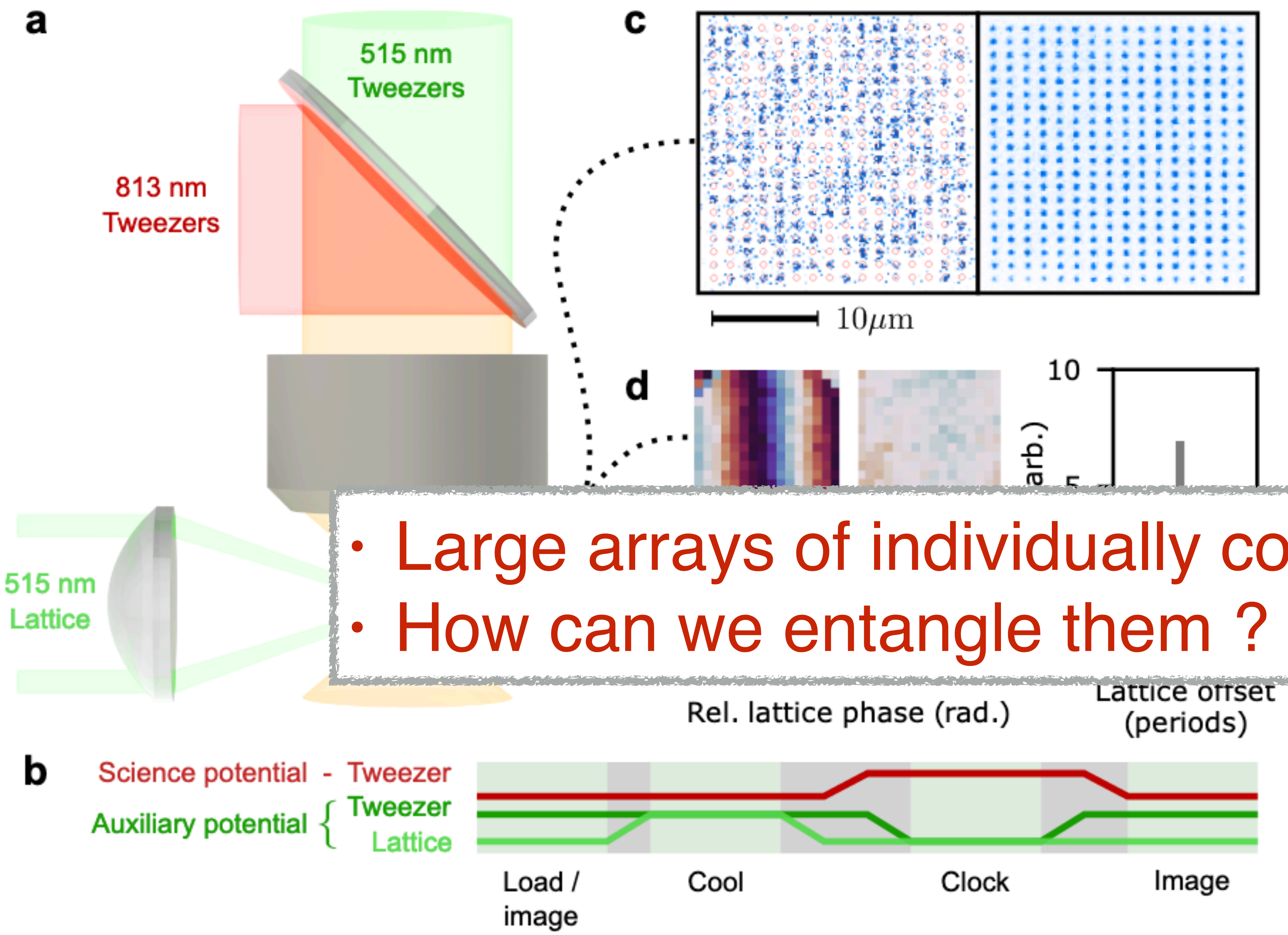


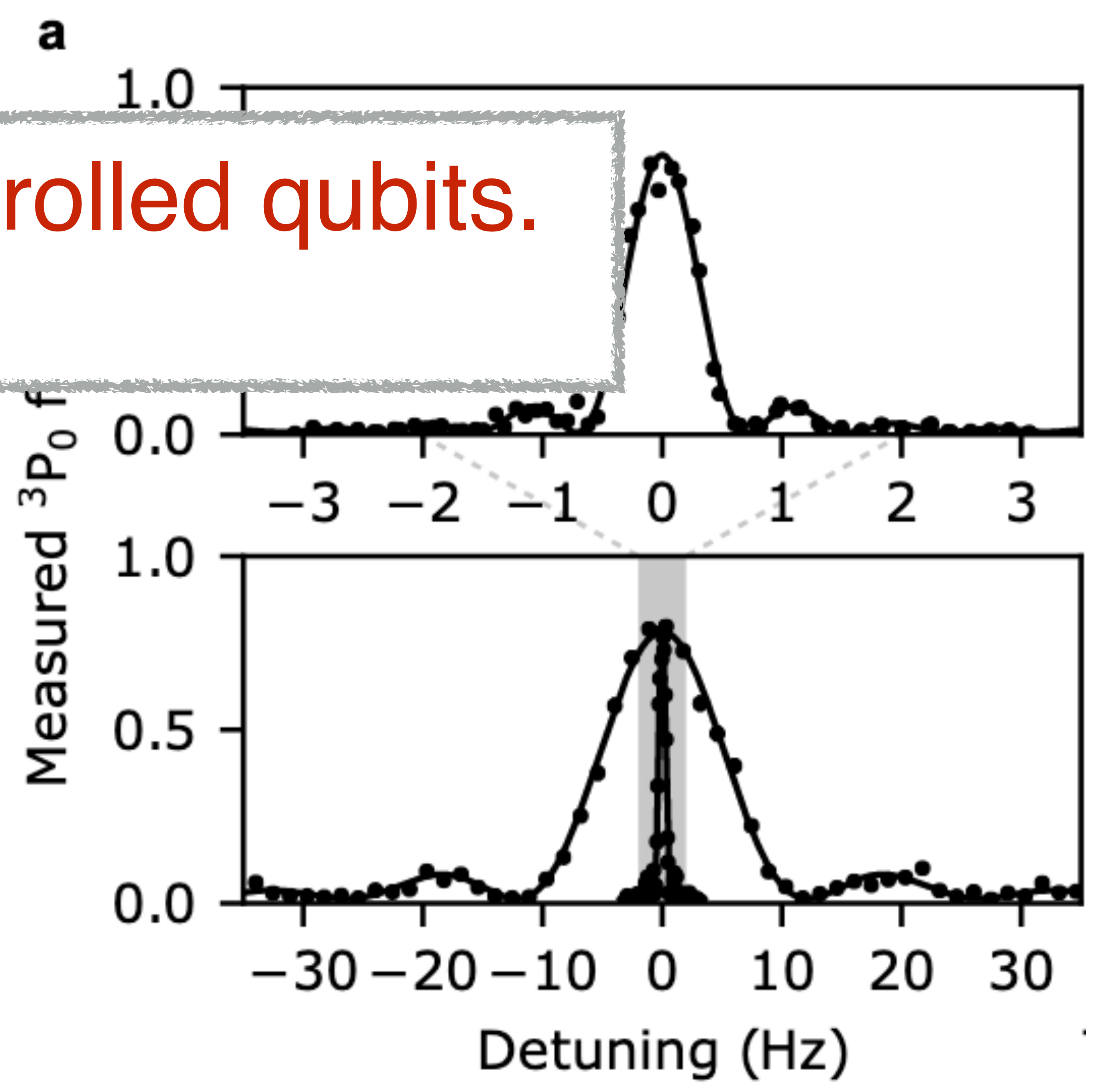


Sub Hz stability on 1000THz transition

Tweezer clocks

A. Young et al. arXiv:2004.06095 (2020)



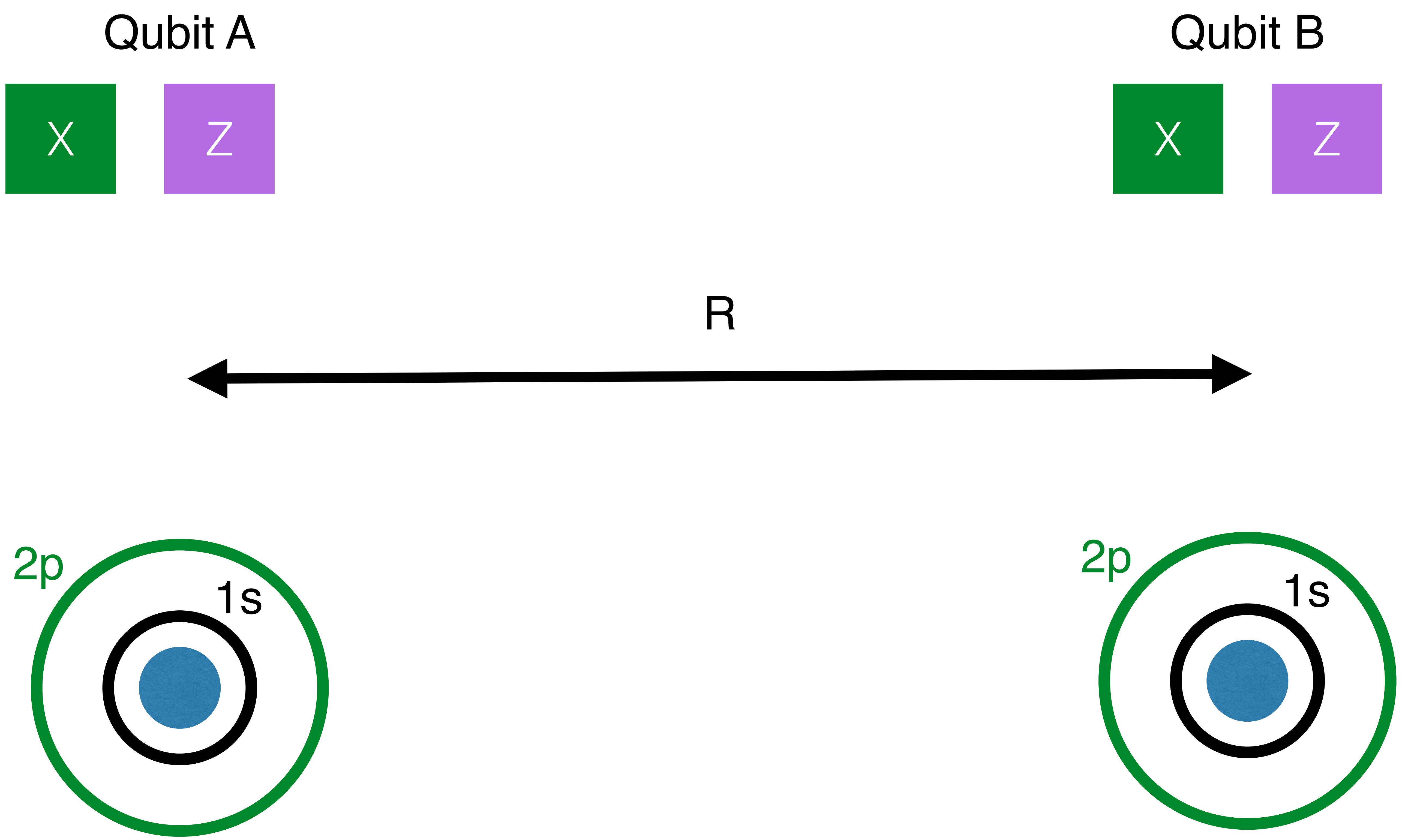


- Large arrays of individually controlled qubits.
- How can we entangle them ?

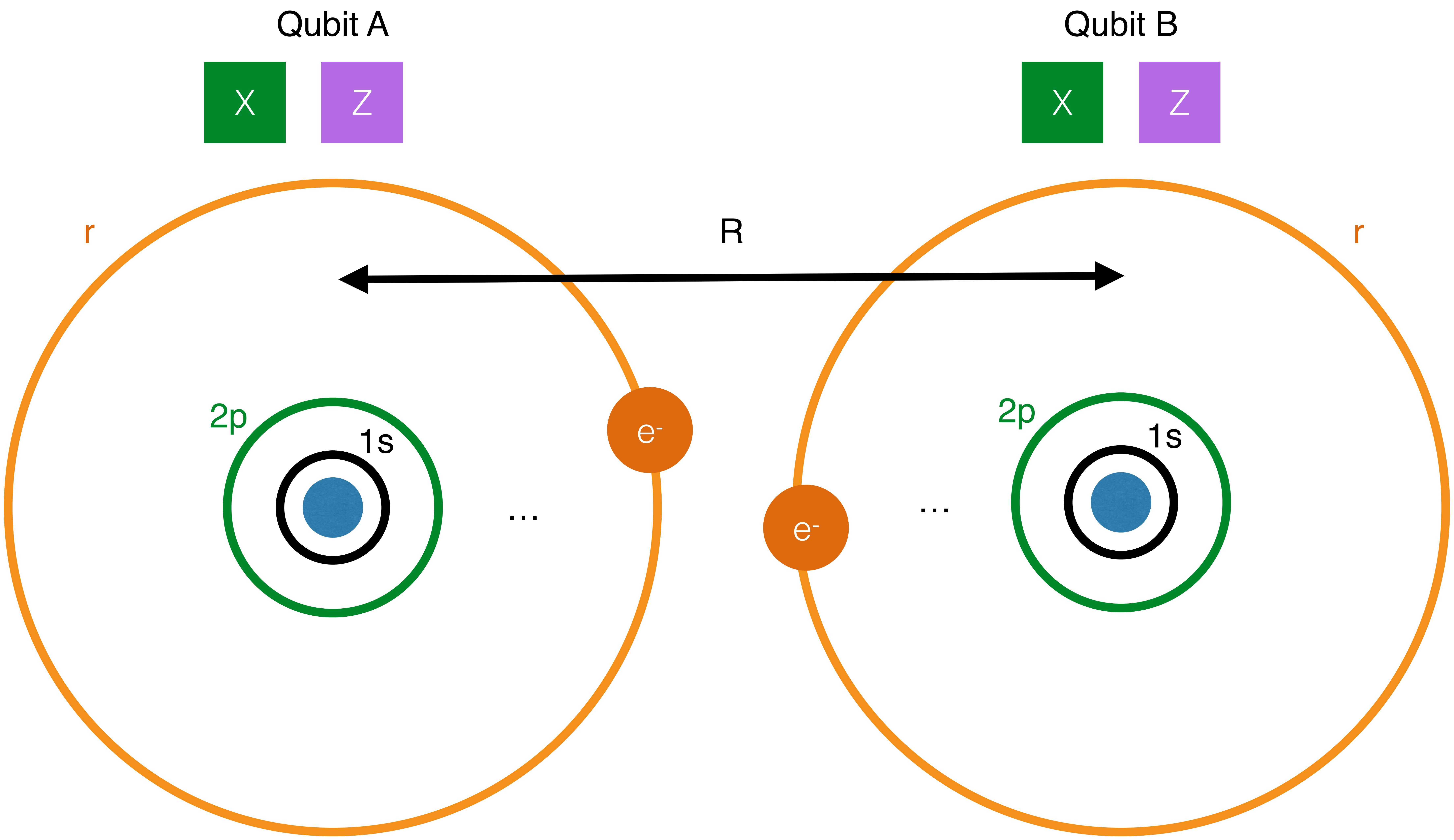
Sub Hz stability on 1000THz transition

1. Atomic clocks — Qubits in cold atoms
2. Optical tweezers — Trapped qubits in atoms
3. Rydberg atoms — Large scale entanglement
4. Moving particles — Bosons vs Fermions and the link to chemistry
5. Lattice gauge theories — Working on a really physics hard problem

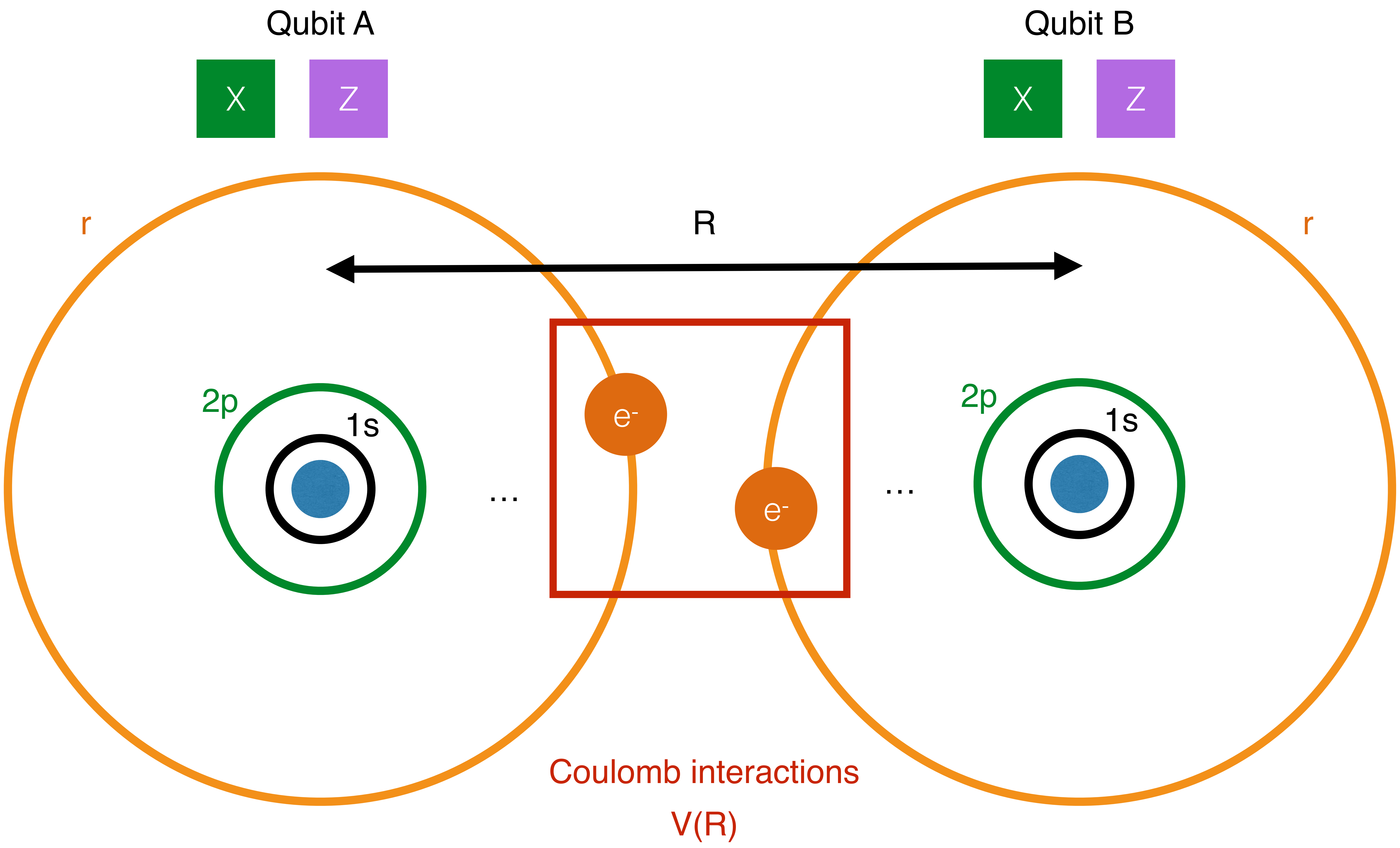
Rydberg atoms - Fast entanglement of neutral atoms



Rydberg atoms - Fast entanglement of neutral atoms

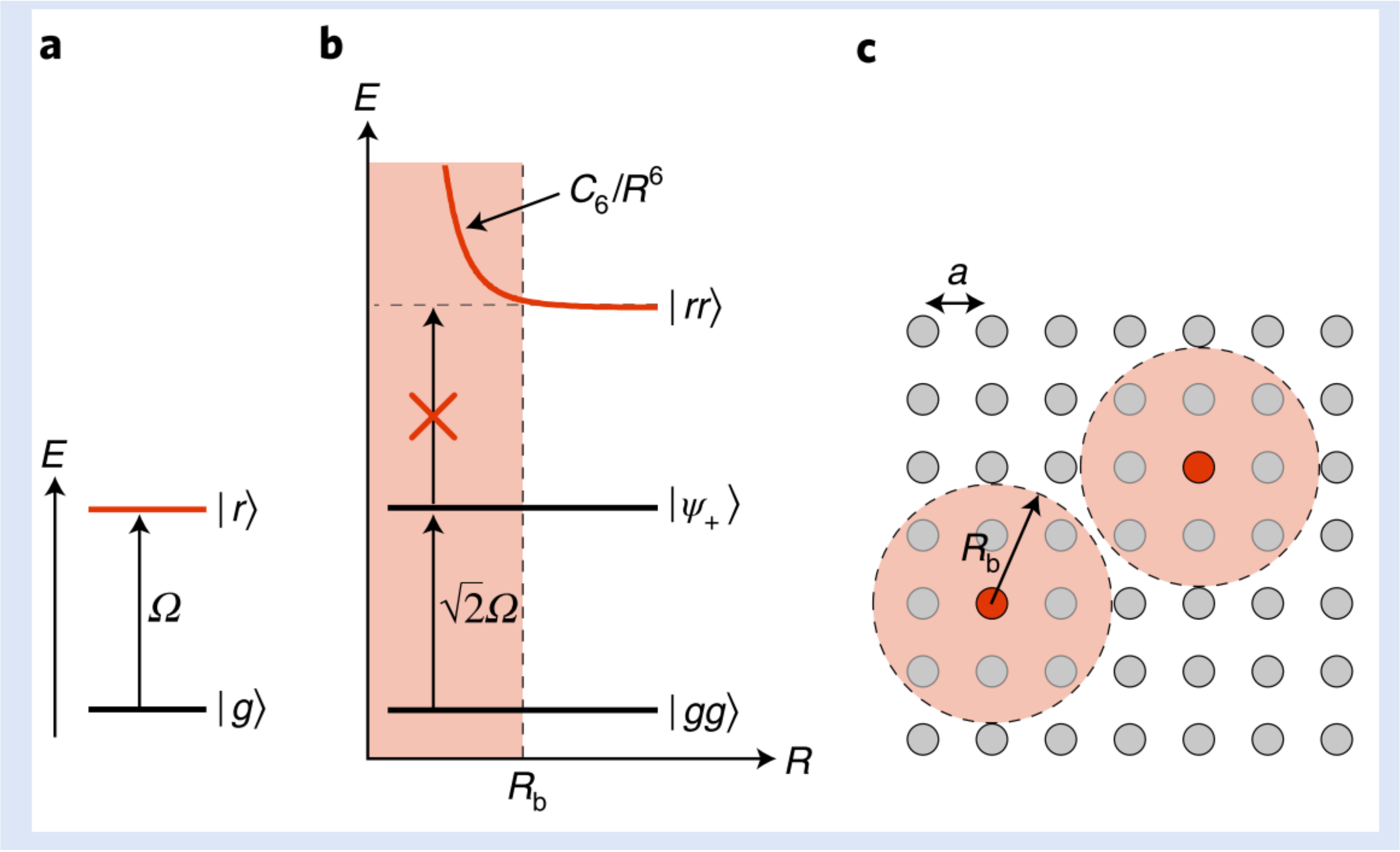


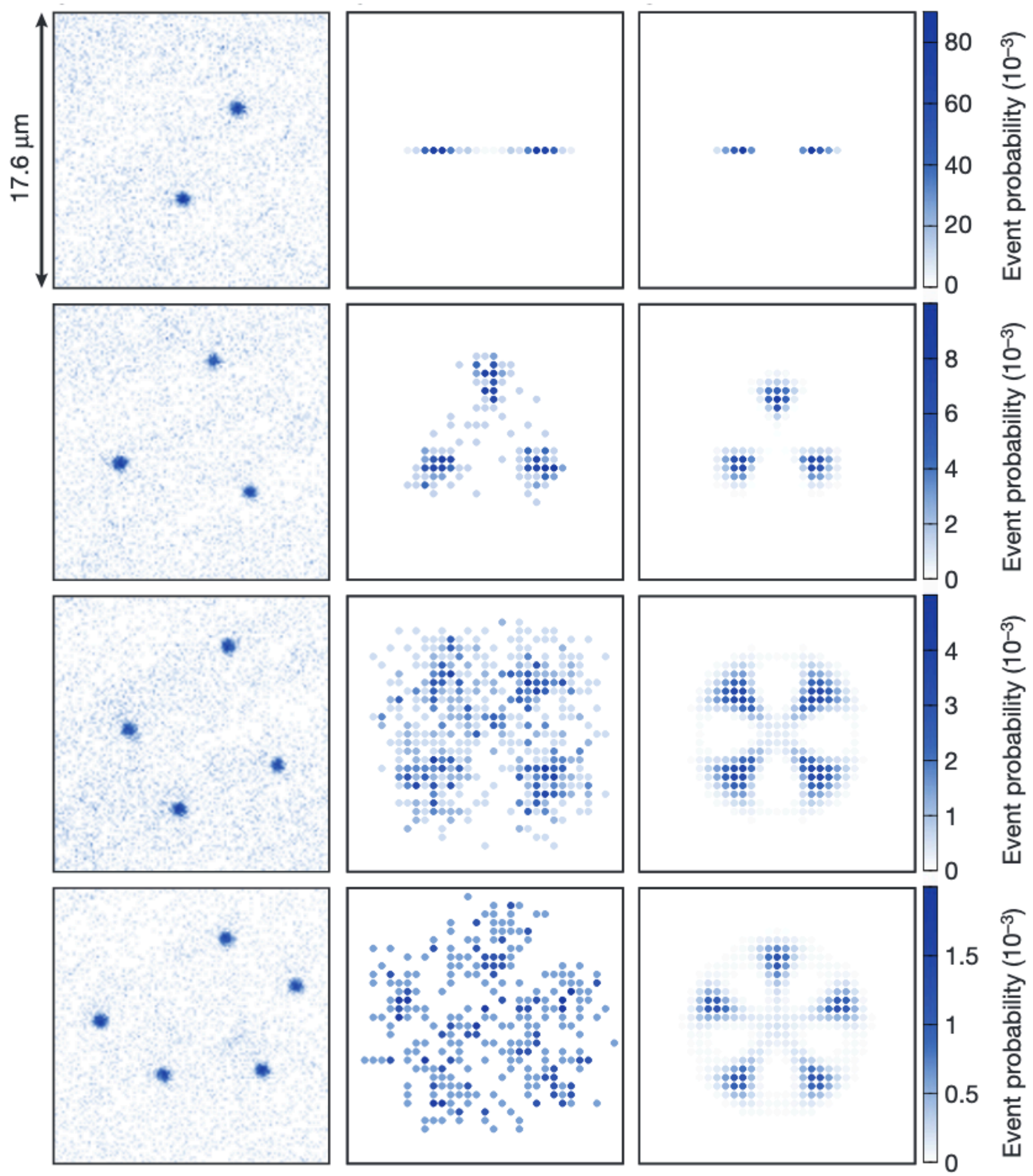
Rydberg atoms - Fast entanglement of neutral atoms



Rydberg atoms - Fast entanglement of neutral atoms

A. Browaeys and T. Lahaye, Nat. Phys. 16, 132 (2020).

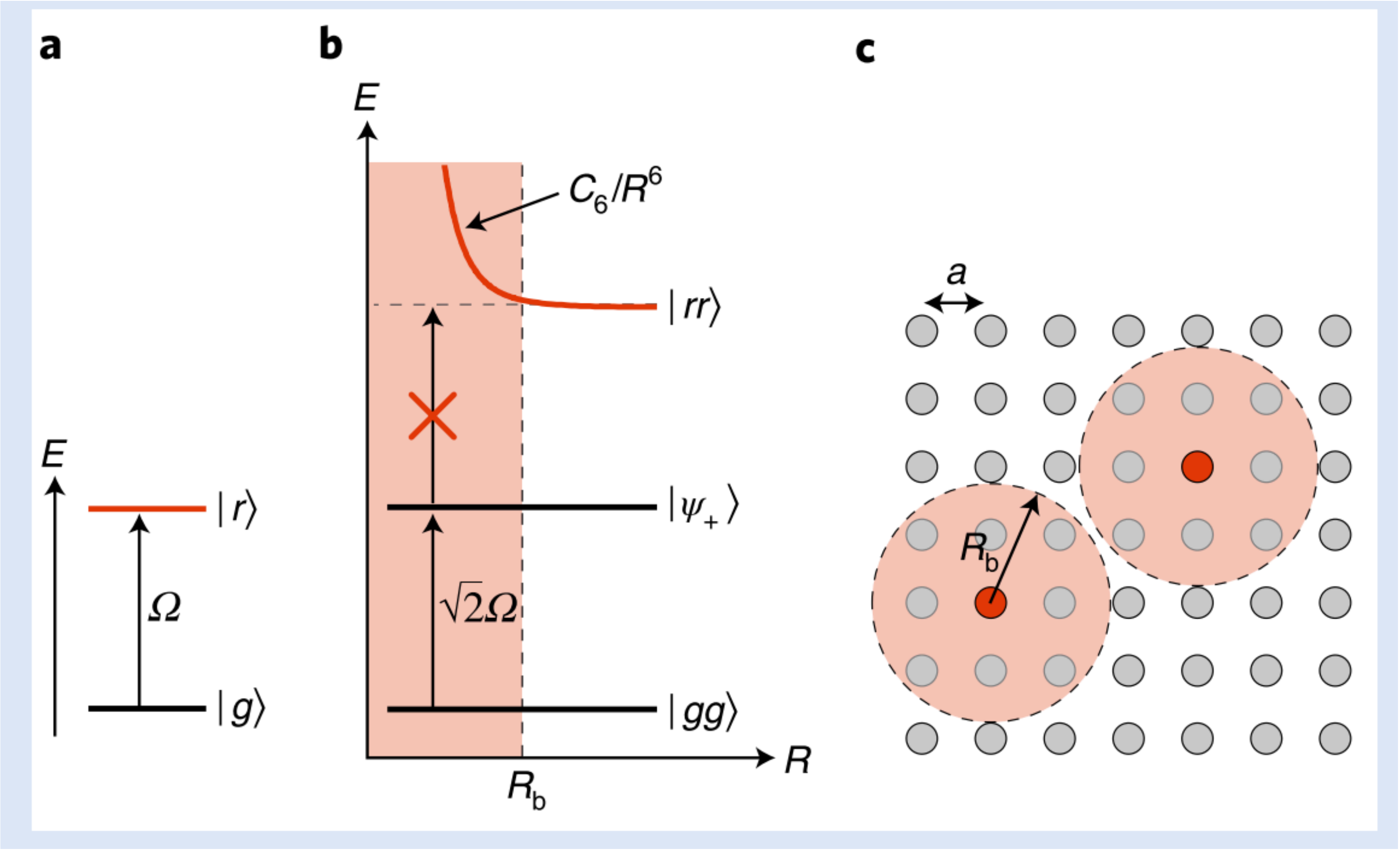


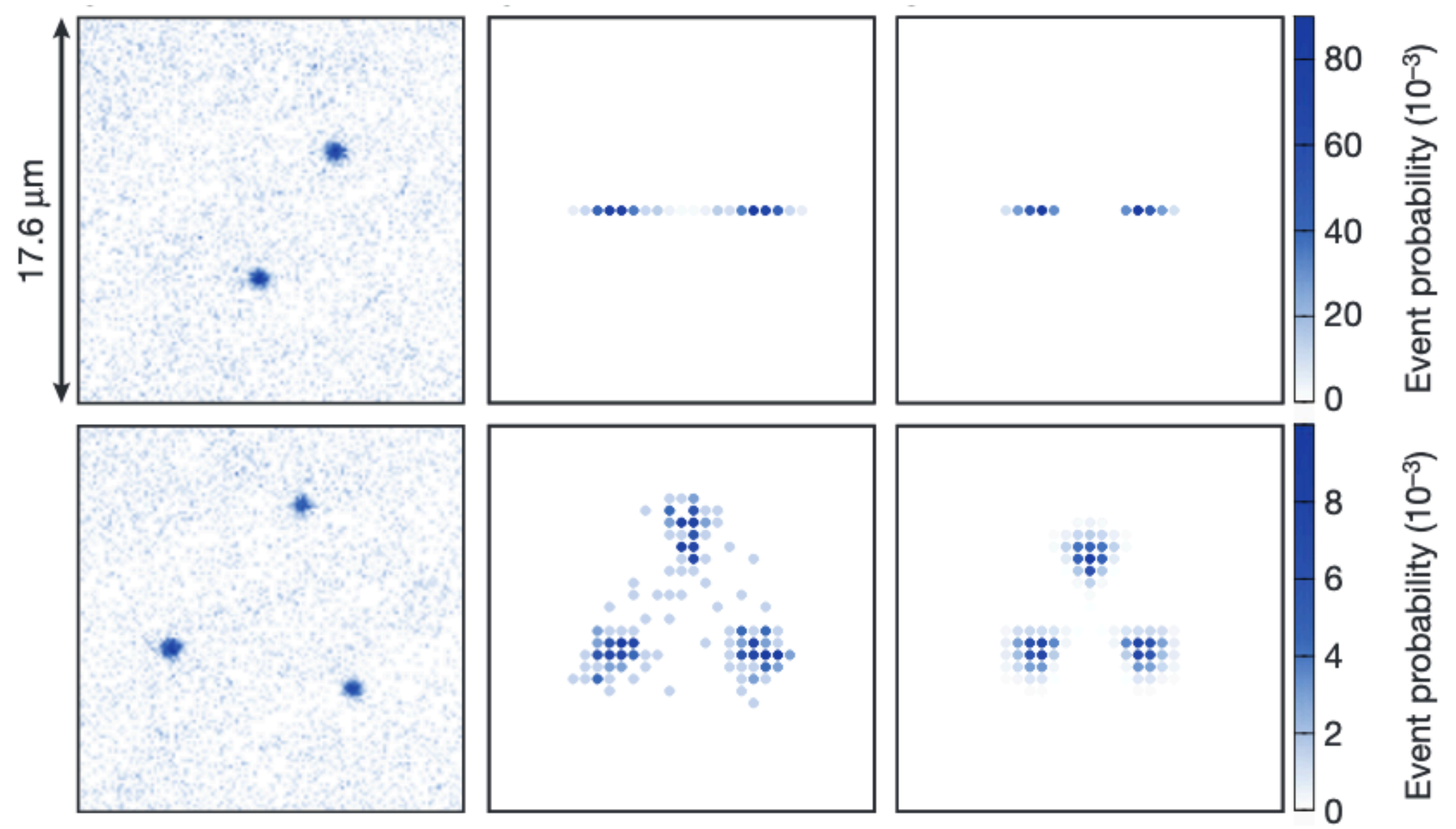


P. Schauß et al. Nature 491, 87 (2012).

Rydberg atoms - Fast entanglement of neutral atoms

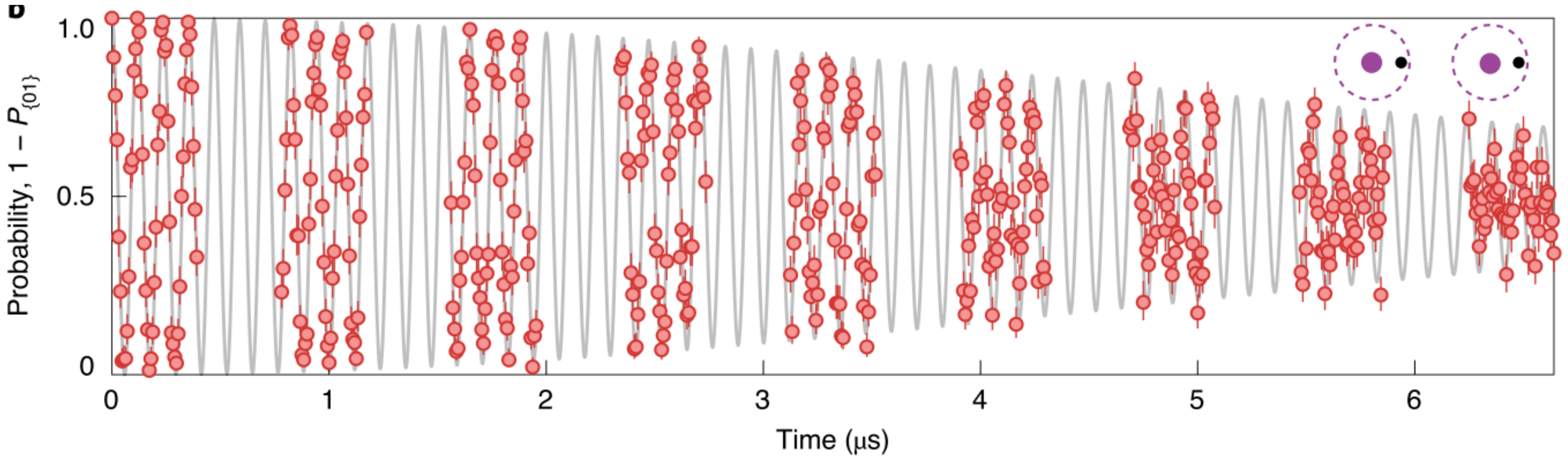
A. Browaeys and T. Lahaye, Nat. Phys. 16, 132 (2020).





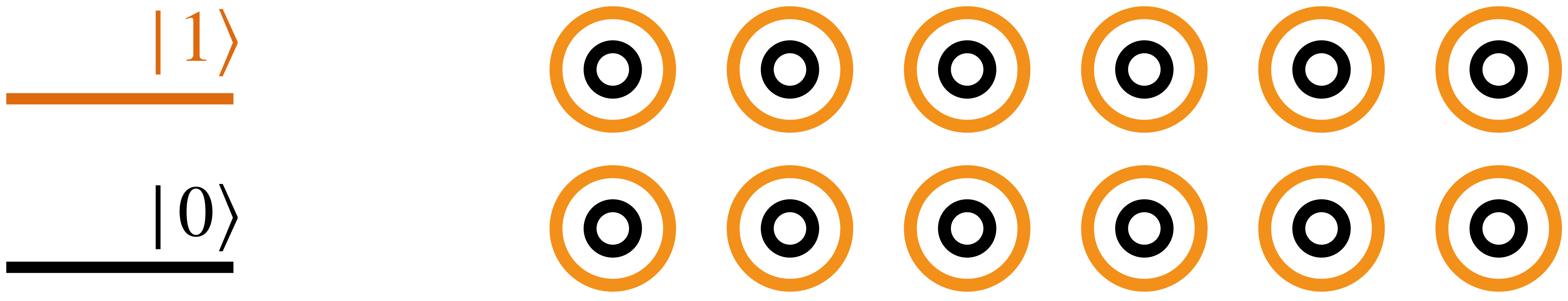
High fidelity entanglement

I. S. Madjarov et al. Nat. Phys. 16, 857 (2020).



Rydberg atoms as quantum simulators

Until now: Use the control to implement a universal gate set



$$\mathcal{H} = \frac{\hbar\Omega}{2}\sum_j \sigma_j^x + \frac{\hbar\Delta}{2}\sum_j \sigma_j^z + \sum_{i\neq j}\frac{C_6}{r_{ij}^6} n_i n_j \qquad n_j = \frac{1}{2} + \sigma_j^z$$

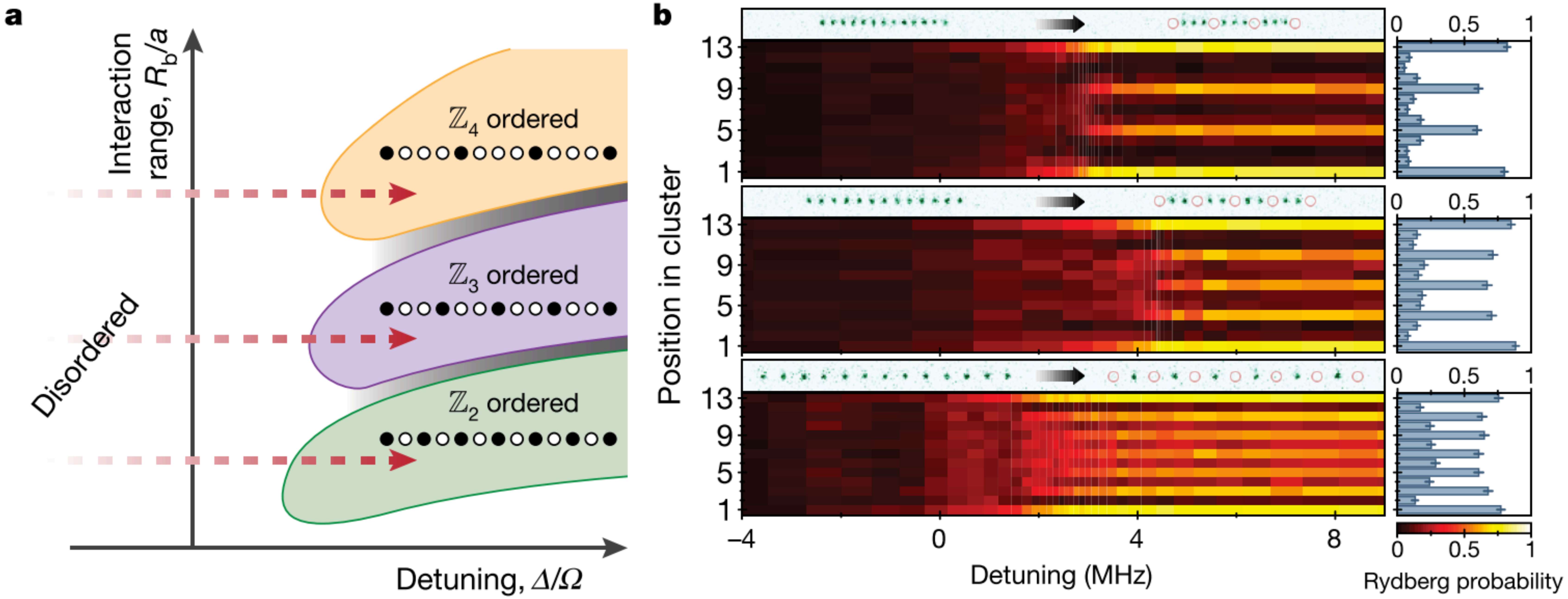
Single qubit control — Two qubit interaction

Now: Use full control over the parameters of the Hamiltonian to solve specific problems.

Rydberg atoms as quantum simulators

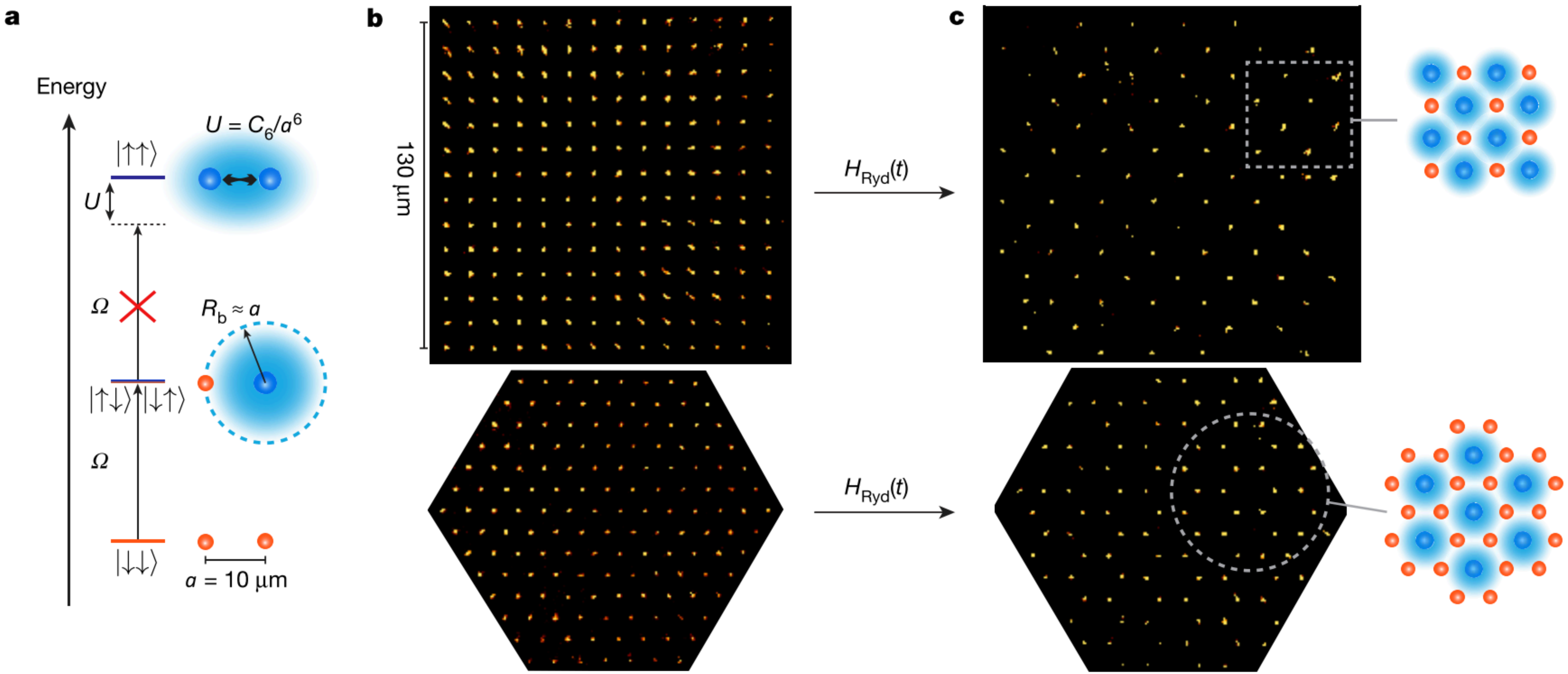
$$\mathscr{H} = \frac{\hbar\Omega}{2}\sum_j \sigma_j^x + \frac{\hbar\Delta}{2}\sum_j \sigma_j^z + \sum_{i\neq j}\frac{C_6}{r_{ij}^6}n_i n_j$$

Quantum simulators with up to 51 atoms



H. Bernien et al. Nature 551, 579 (2017).

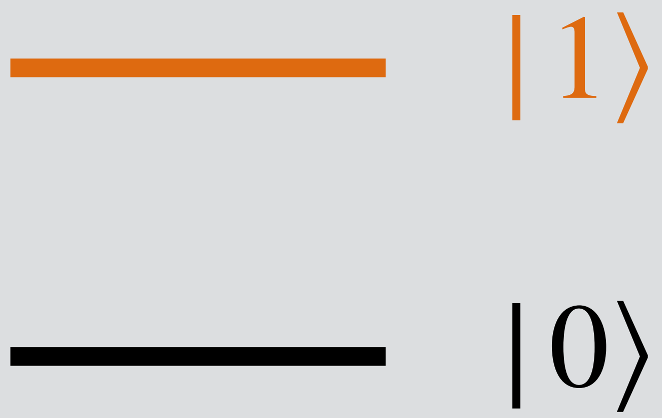
Rydberg simulators in 2D



P. Scholl et al. Nature, 595, 233 (2021)

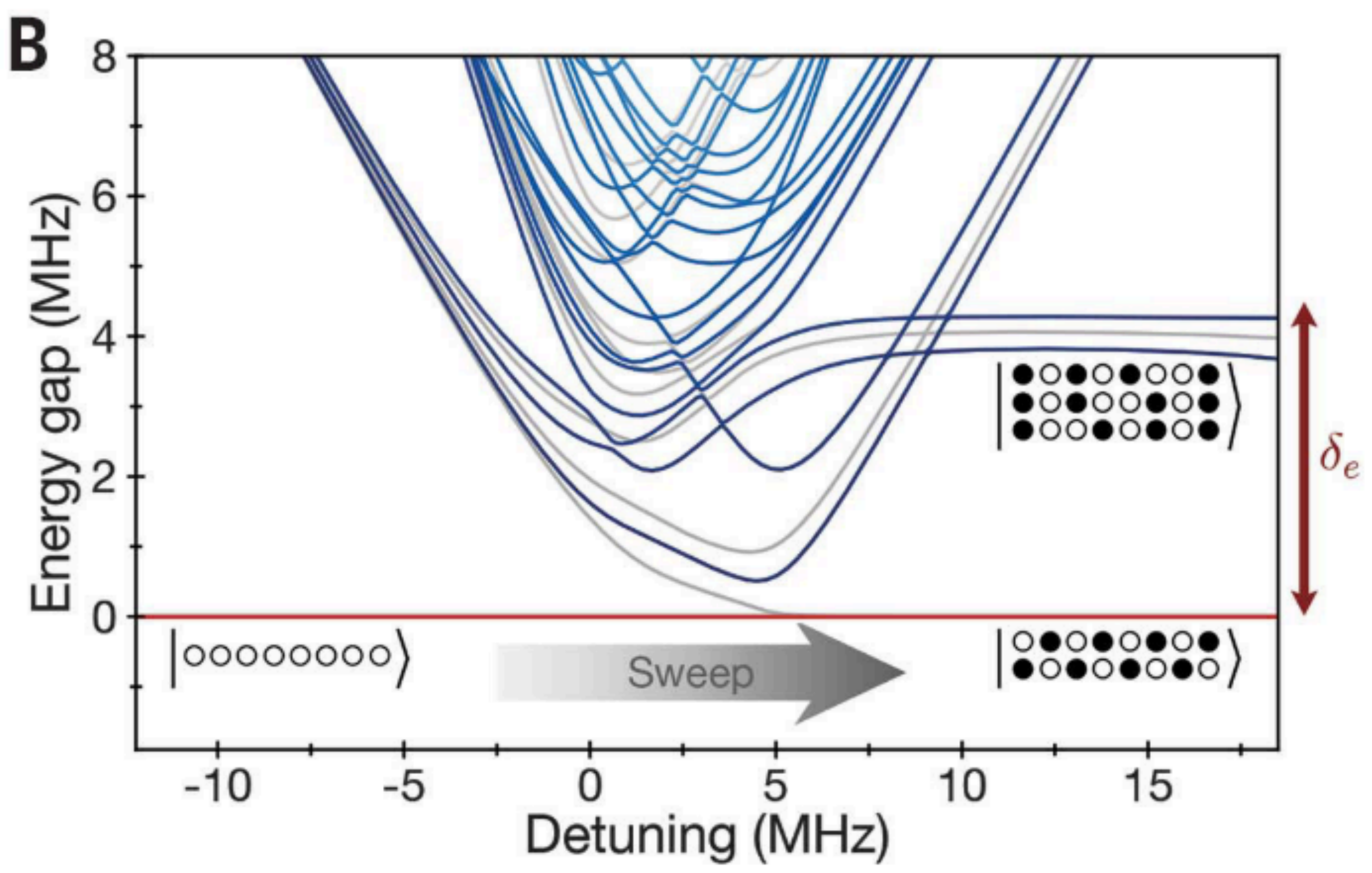
Ebadi, S. et al. Nature, 595, 227 (2021).

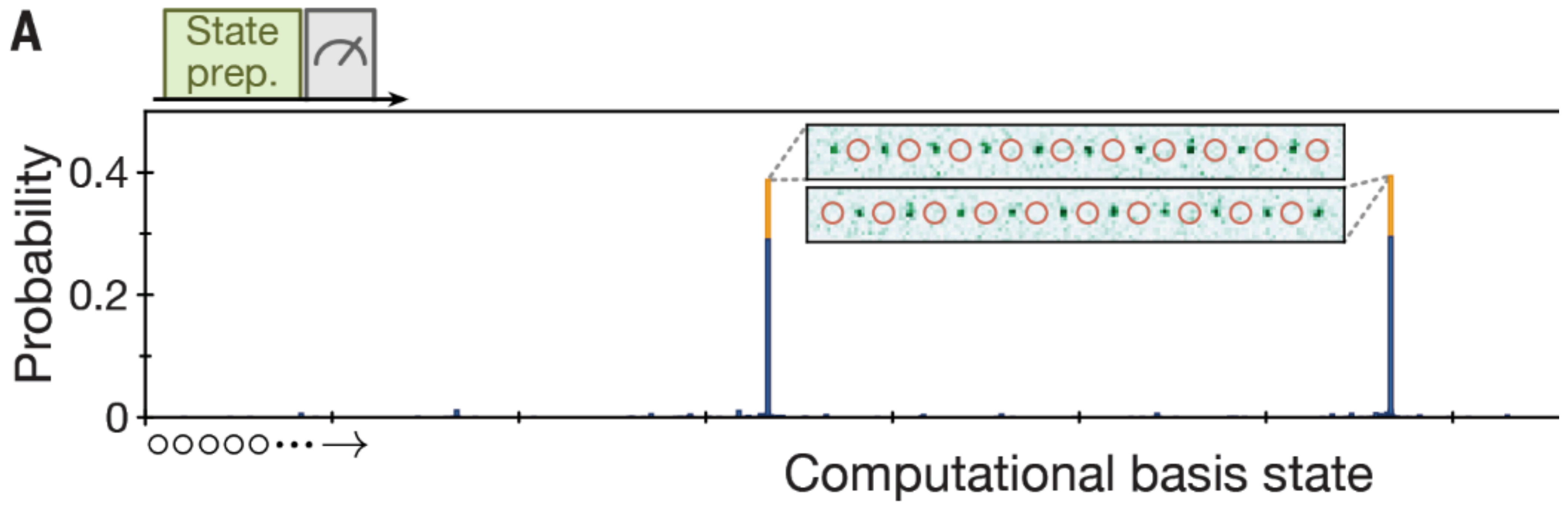
Rydberg atoms as quantum simulators



$$\mathscr{H} = \frac{\hbar\Omega}{2}\sum_j \sigma_j^x + \frac{\hbar\Delta}{2}\sum_j \sigma_j^z + \sum_{i\neq j}\frac{C_6}{r_{ij}^6}n_i n_j$$

Schrödinger cats with 20 atoms



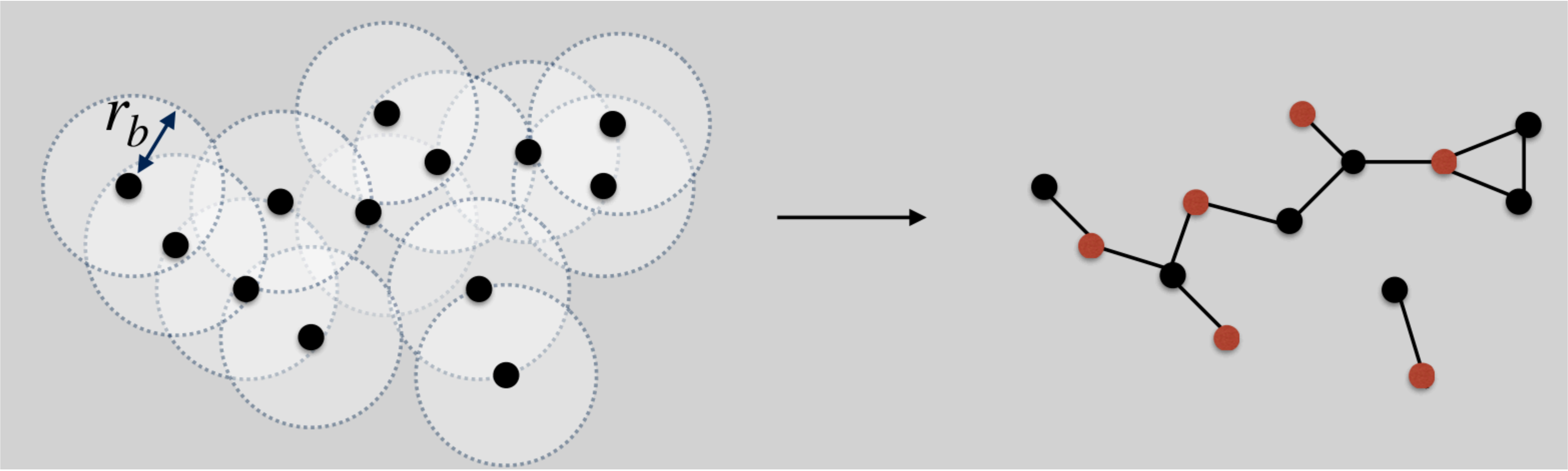


A. Omran et al. Science 365, 570 (2019).

Maximum Independent Sets

L. Henriet et al. Quantum 4, 327 (2020).

H. Pichler et al., ArXiv 1808.10816 (2018).



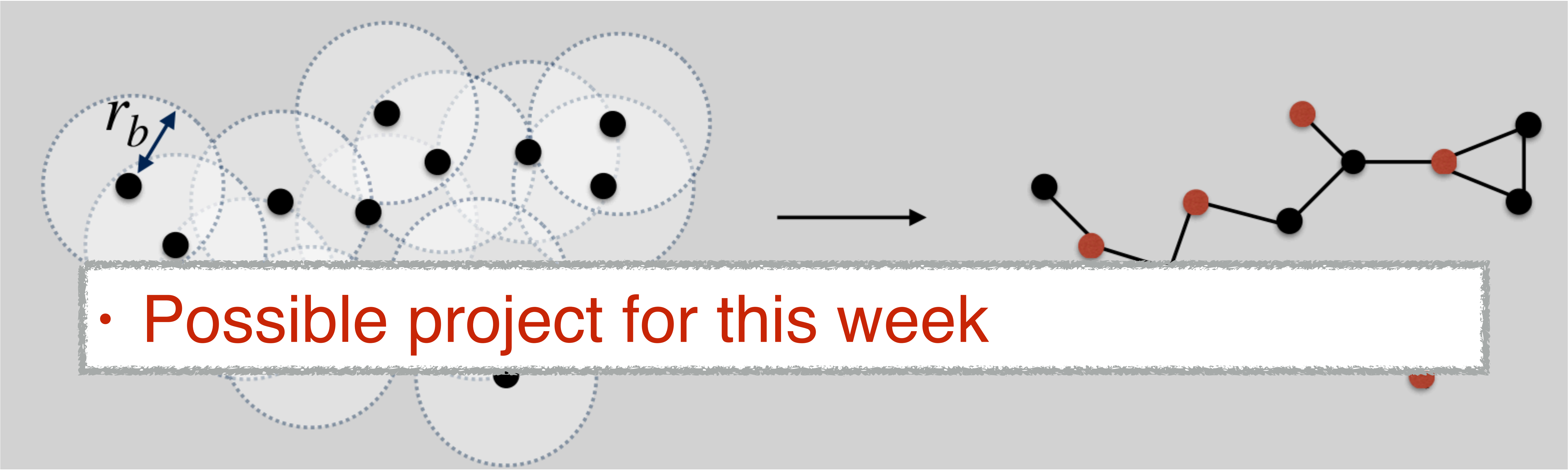
Possible applications to finance, network design

|QuEra⟩ atom computing PASQAL

Maximum Independent Sets

L. Henriet et al. Quantum 4, 327 (2020).

H. Pichler et al., ArXiv 1808.10816 (2018).



- Possible project for this week

Possible applications to finance, network design

Digital vs analog

	Analog Quantum Simulation	**Digital Quantum Simulation**
Resource used for simulation	Hamiltonians	Gates
Key advantages	Promising hybrid quantum-classical approaches	Universal approach
Shortcomings	Limited number of available configurations	Requires a large number of gates
Status	Quantum advantage already achieved	Academic research

L. Henriet et al. Quantum 4, 327 (2020).

1. Atomic clocks — Qubits in cold atoms
2. Optical tweezers — Trapped qubits in atoms
3. Rydberg atoms — Large scale entanglement
4. Moving particles — Bosons vs Fermions and the link to chemistry
5. Lattice gauge theories — Working on a really physics hard problem

What happens when you cool a gas?

Average velocity of thermal atoms:

$$\langle |v| \rangle = \sqrt{\frac{3k_BT}{m}}$$

atoms = wave packets

$$\lambda_{dB} = \frac{h}{mv} = \sqrt{\frac{h^2}{3mk_BT}}$$

Thermal de Broglie wavelength

High Temperature T:

thermal velocity v

density d^{-3}

"Billiard balls"

Low Temperature T:

De Broglie wavelength

$\lambda_{dB}=h/mv \propto T^{-1/2}$

"Wave packets"

$T=T_{crit}$: Bose-Einstein Condensation

$\lambda_{dB} \approx d$

"Matter wave overlap"

T=0: Pure Bose condensate

"Giant matter wave"

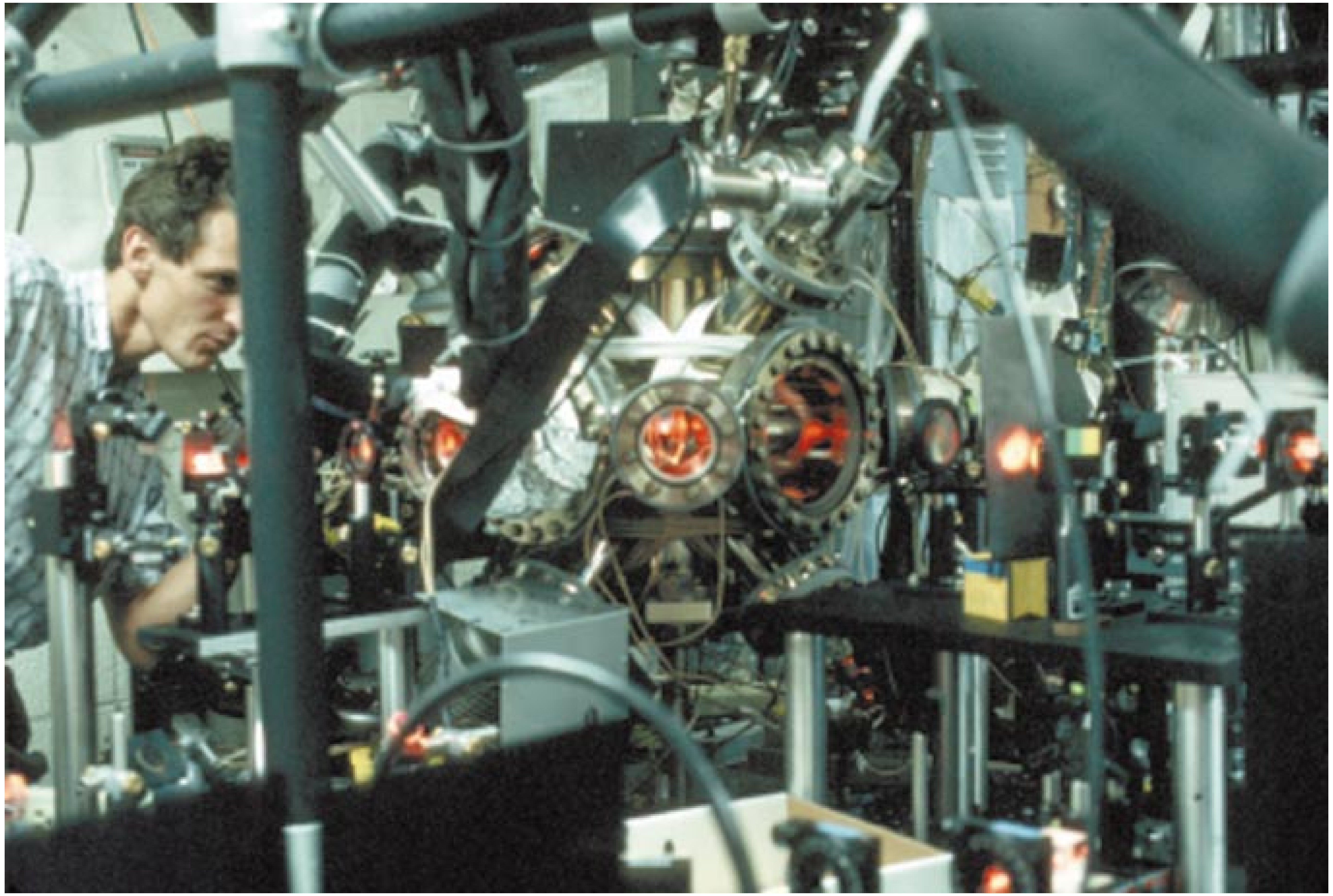
How to get colder?

Evaporative Cooling

Hess, PRB **34** 3476 (1985)

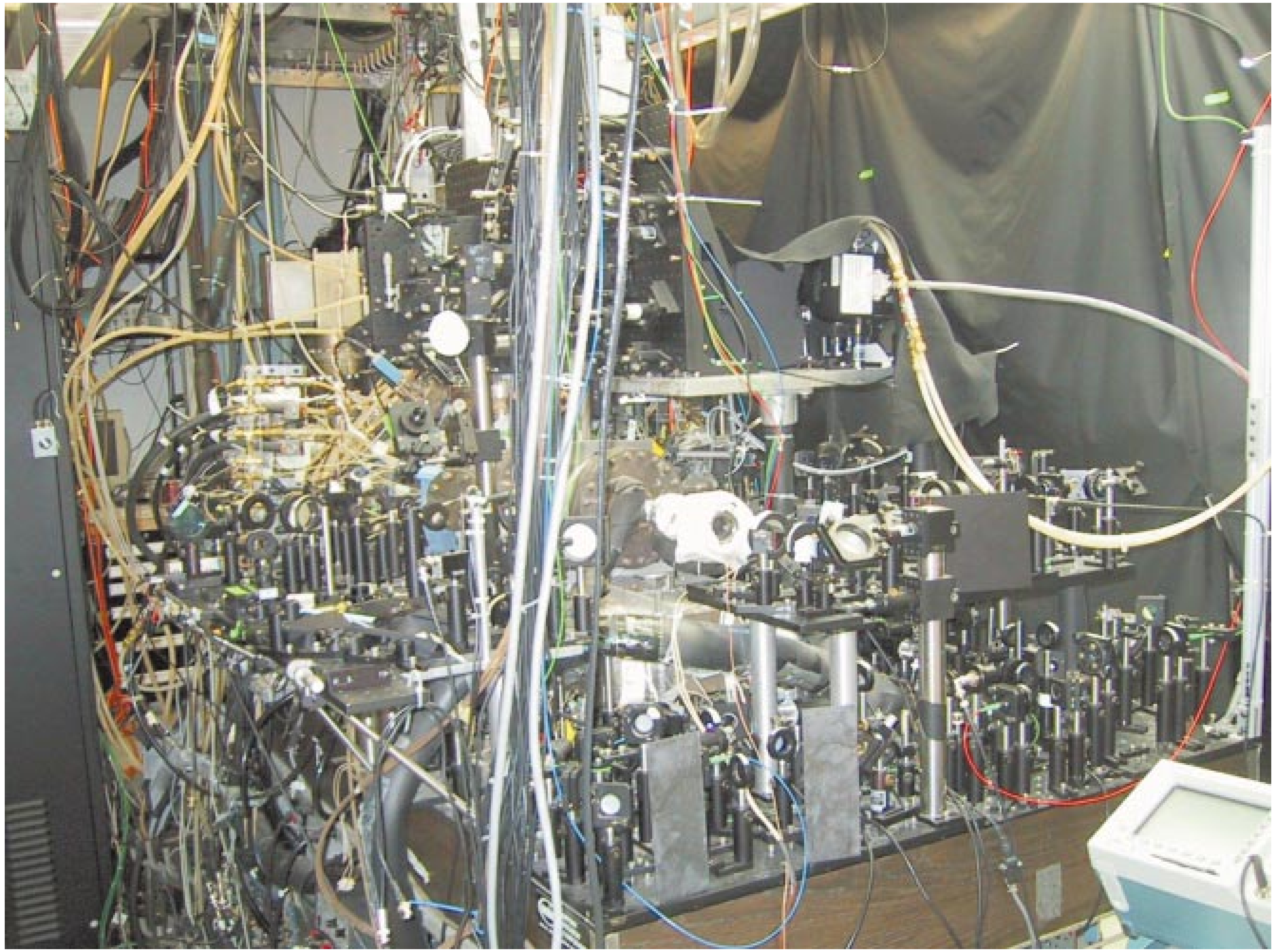
How do you create a BEC ?

before BEC



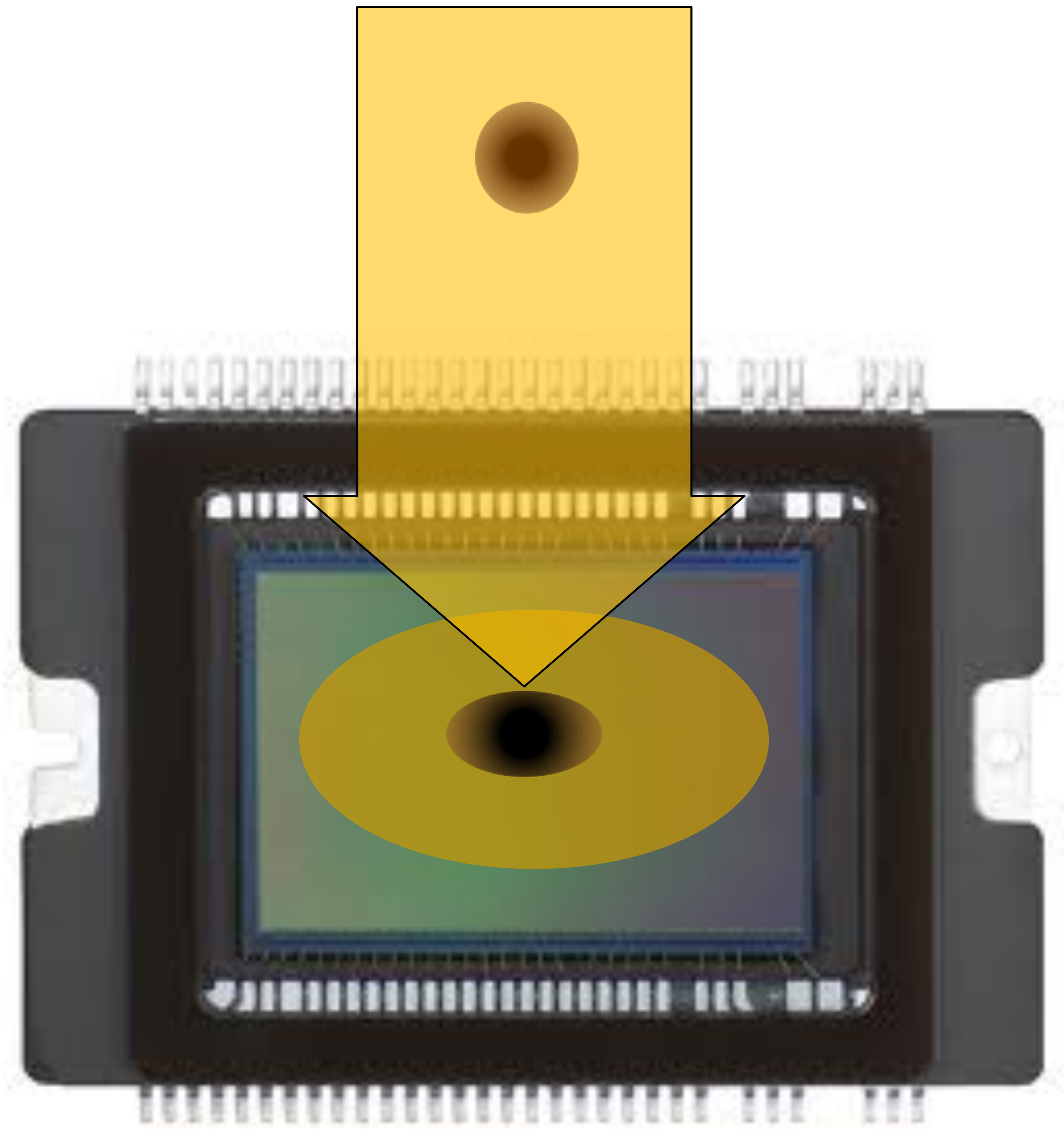
How do you create a BEC ?

with BEC



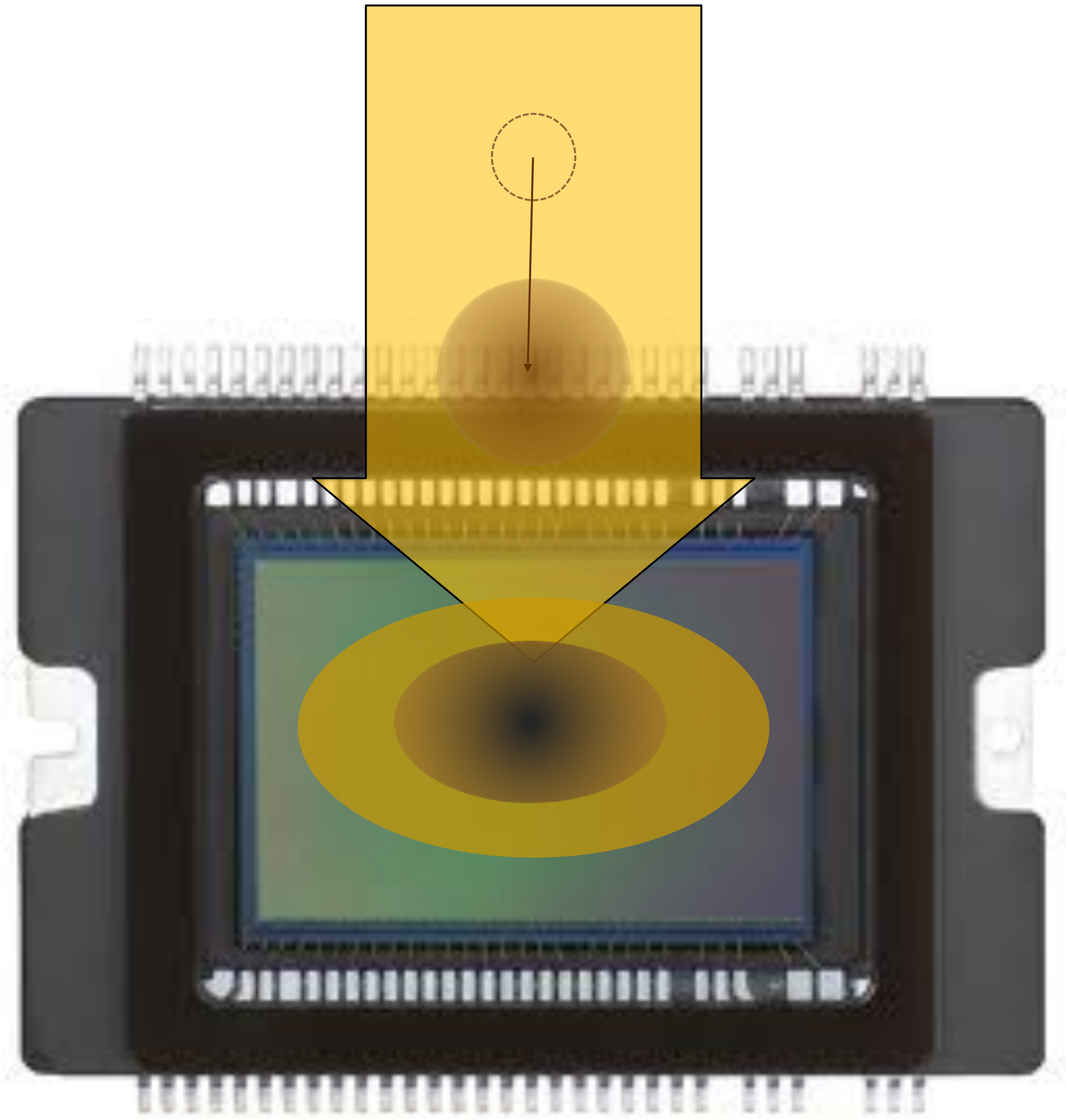
Absorption Imaging

In-situ



Spatial Information

Time-of-Flight (TOF)



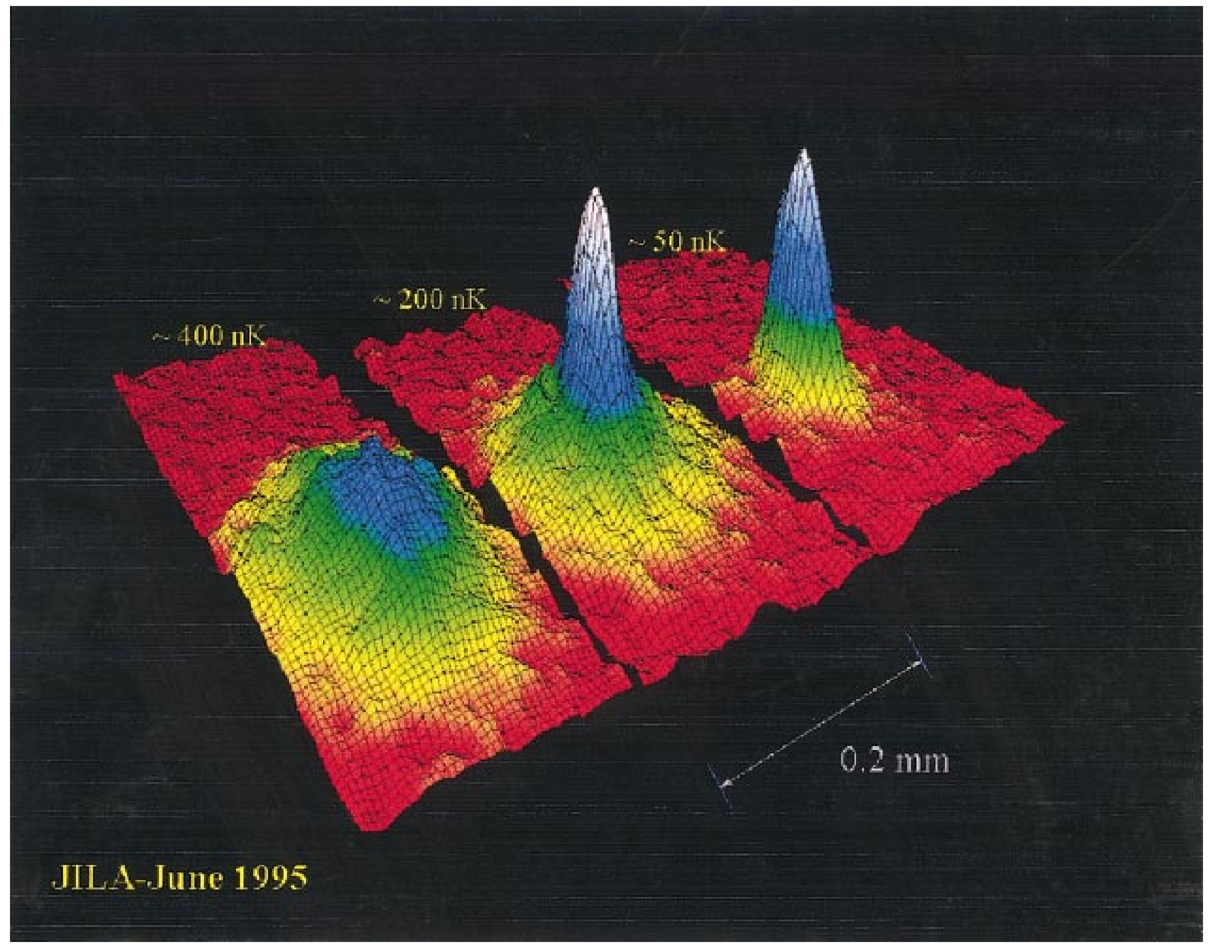
Momentum Information

How do you see a BEC ?

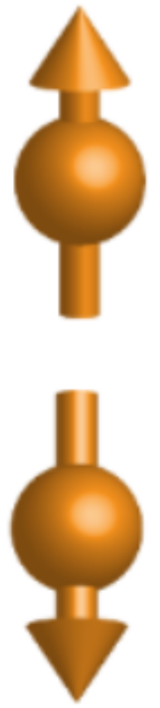
Initial trap (Real space):

<u>Time of flight</u>
cut the trap and
let it expand freely

Momentum distribution:



single atom — qubit — $\ell = 1/2$



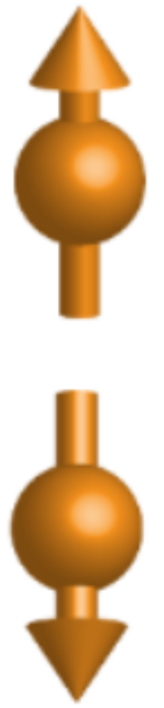
$m_F = 1$

$m_F = 0$

$$|\psi\rangle = \alpha_{-1/2}|-1/2\rangle + \alpha_{1/2}|1/2\rangle$$

$\hat{L}_z|n\rangle = n|n\rangle$ with integer $|n| \leq \ell$

$$[\hat{L}_x, \hat{L}_y] = i\hat{L}_z$$



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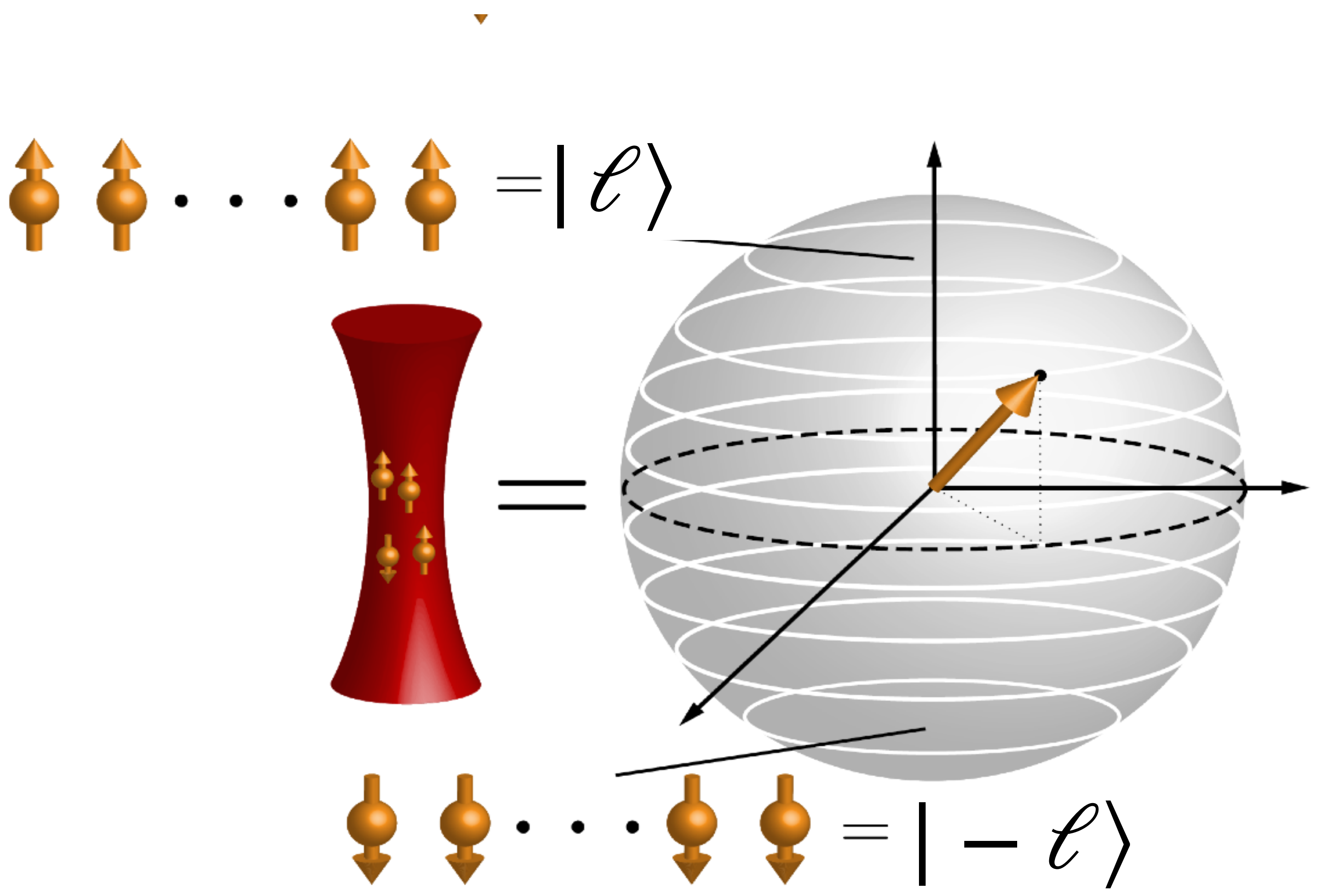
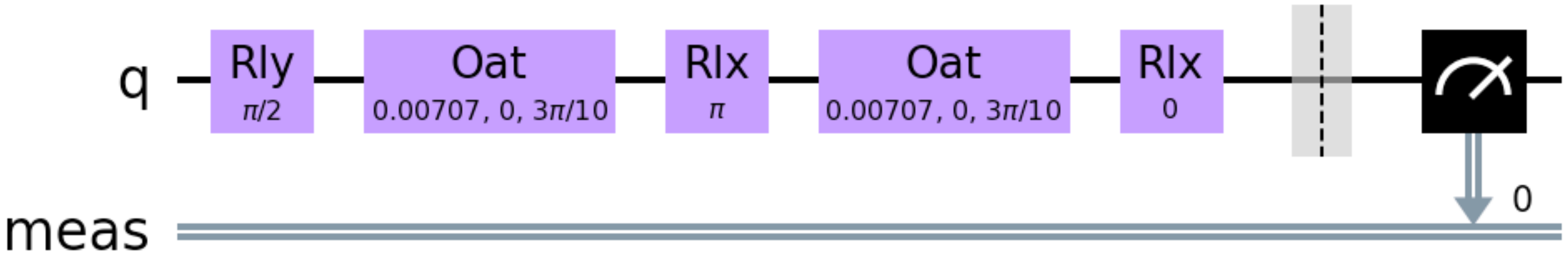
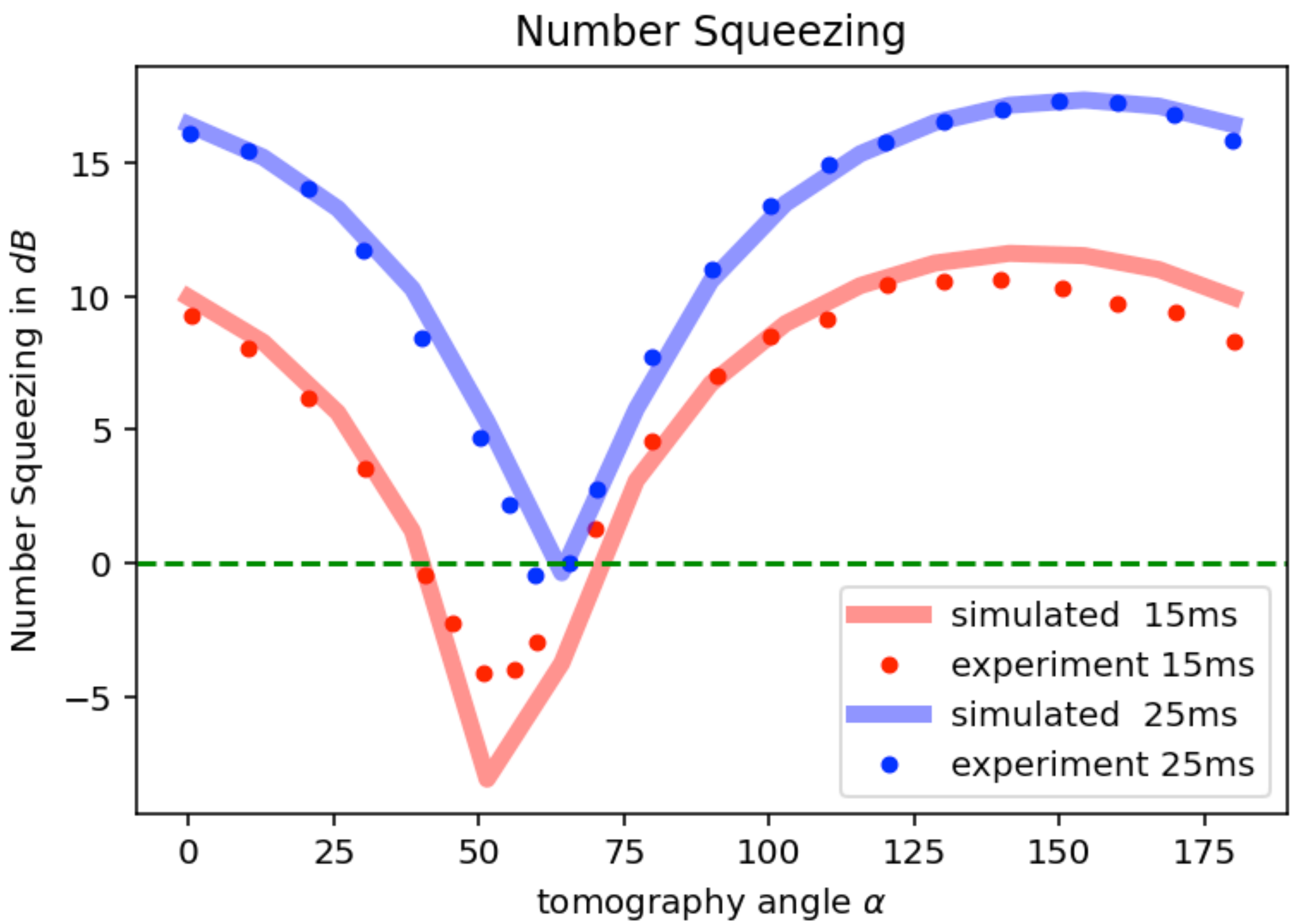


N indistinguishable atoms — qudit — $\ell = N/2$

$$|\psi\rangle = \alpha_{-\ell}|-\ell\rangle + \cdots + \alpha_{\ell}|\ell\rangle$$

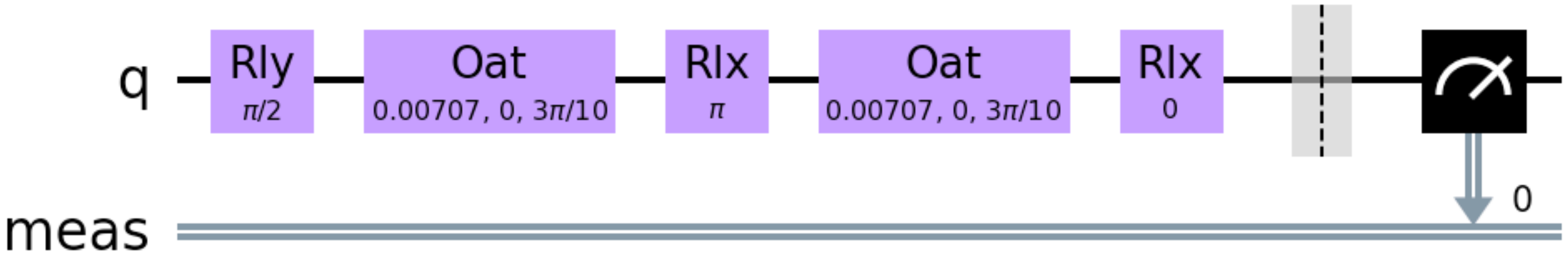
Squeezing with cold atoms

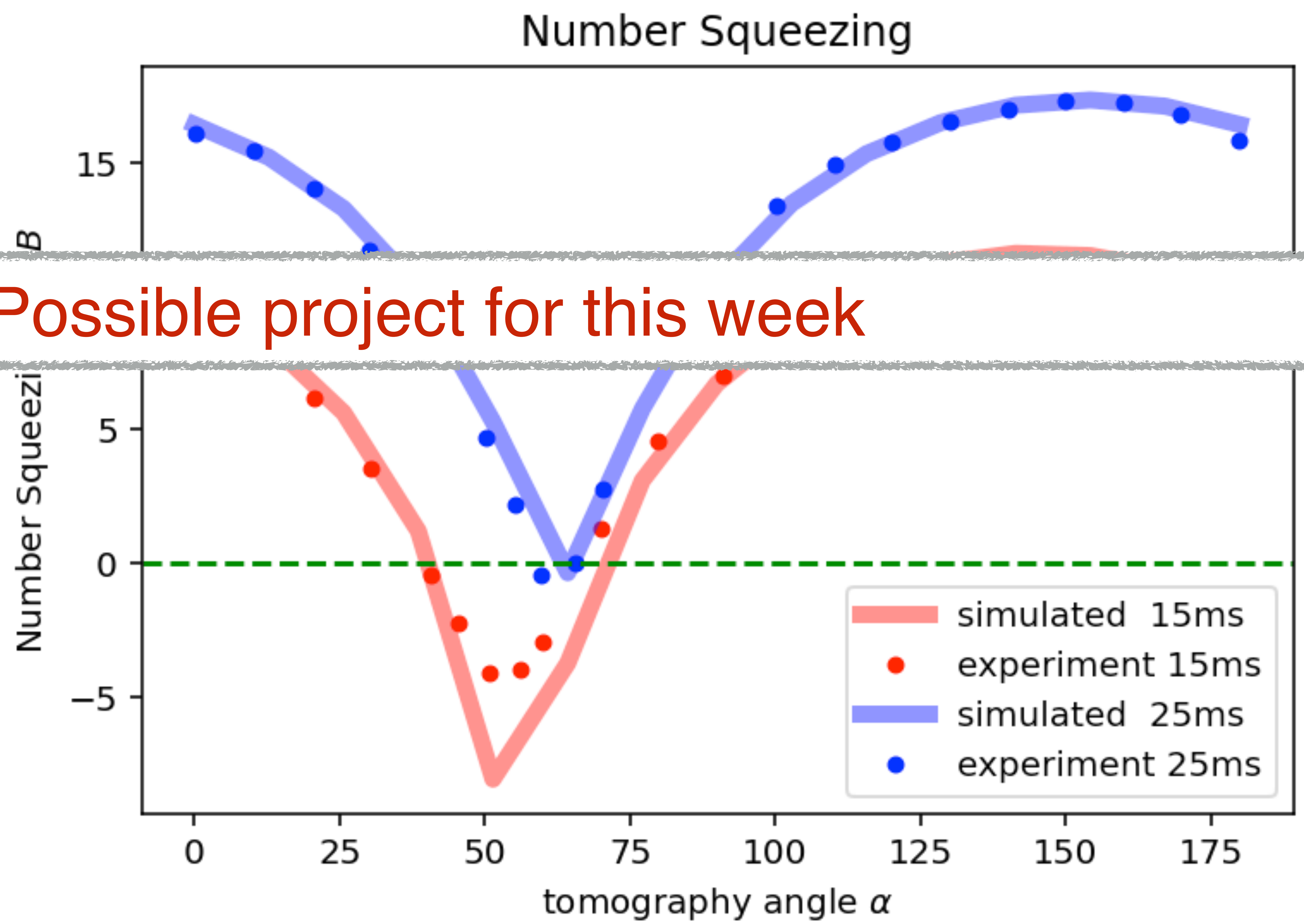




H. Strobel et al. Science, 345, 424 (2014)

Squeezing with cold atoms

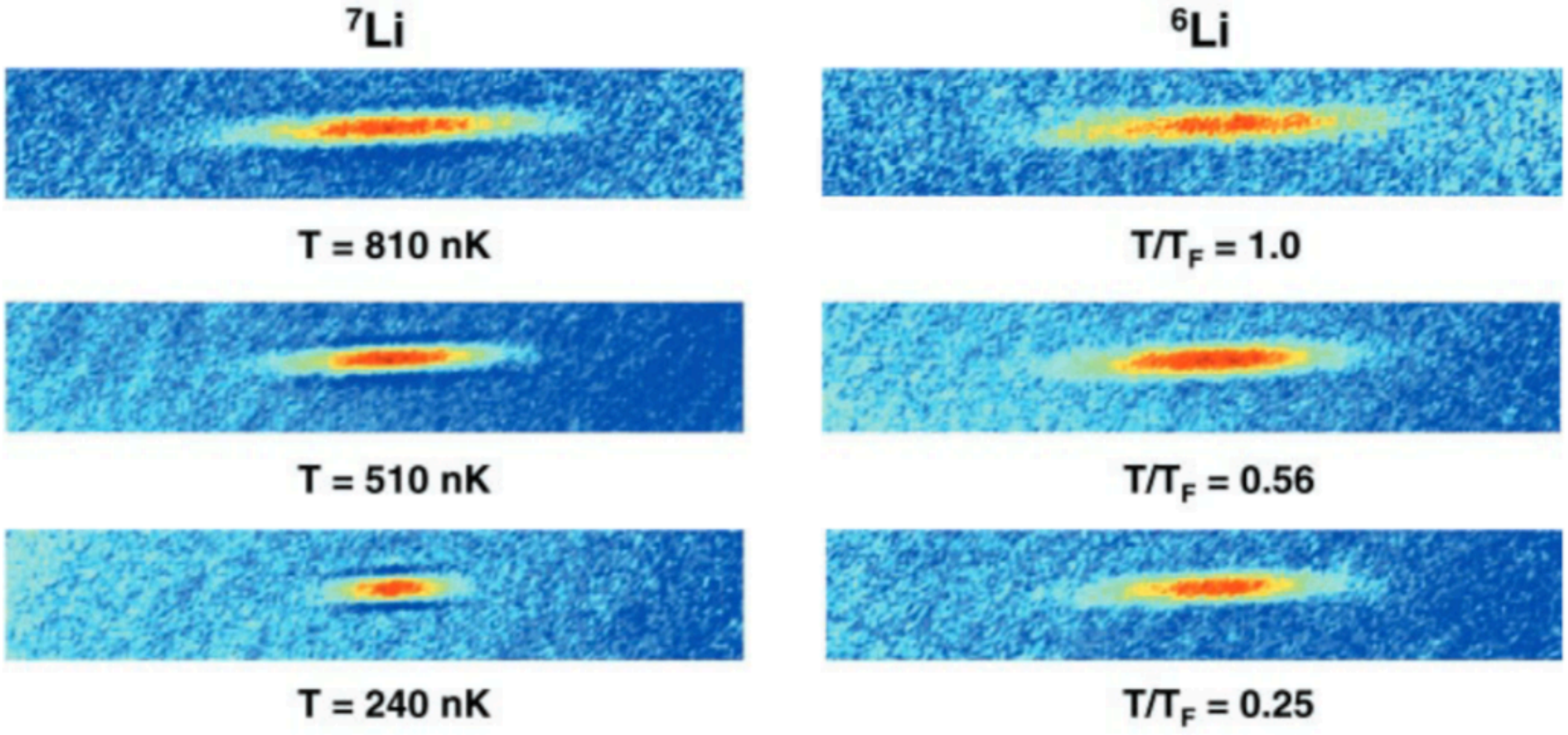




- Possible project for this week

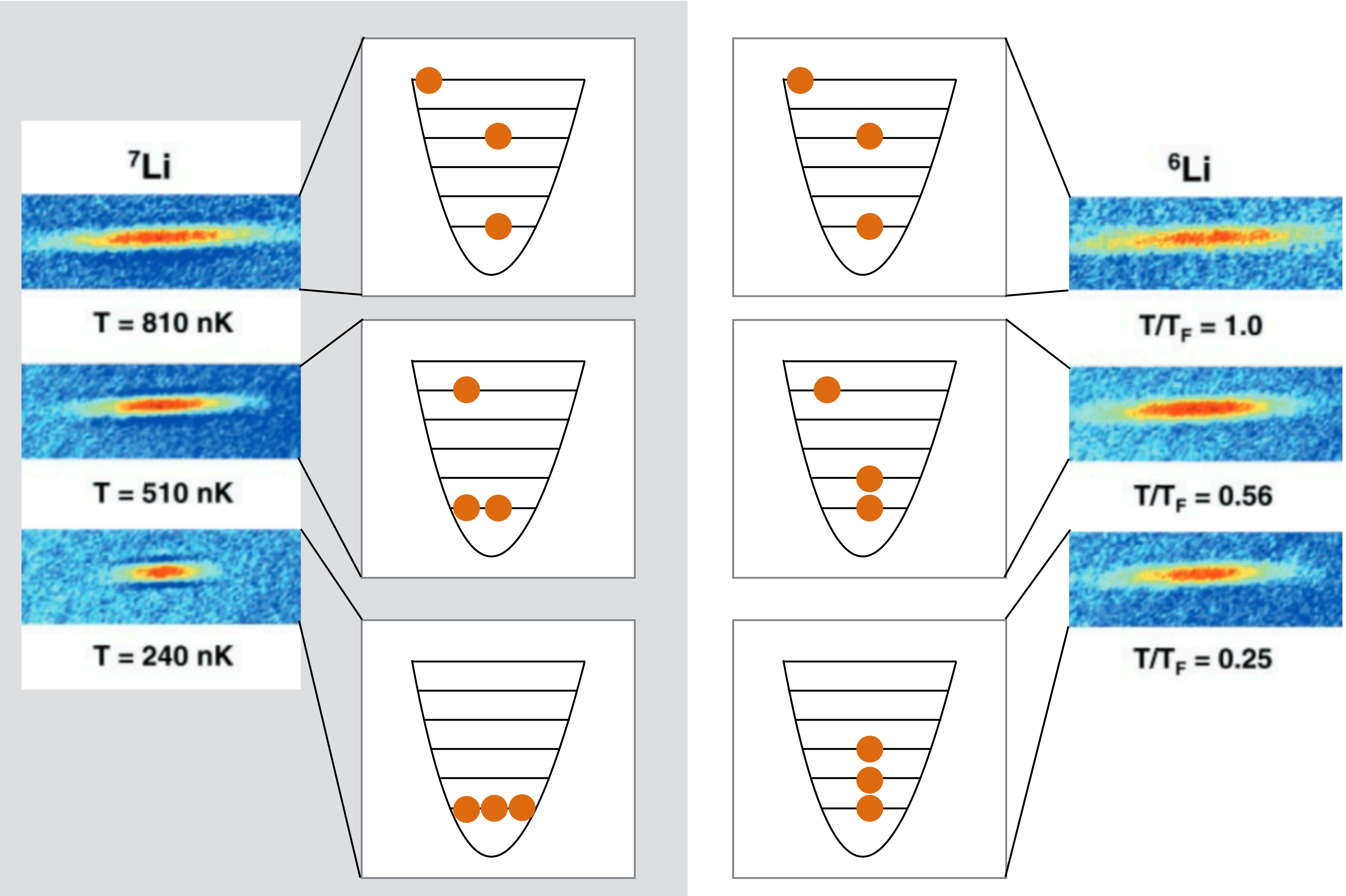
H. Strobel et al. Science, 345, 424 (2014)

On fermion vs bosons



A. G. Truscott et al. Science 291, 2570 (2001).

On fermion vs bosons



Second quantization

Many particles: full wave-function has to be properly symmetrized

Bosons

$$\psi(1,2) = \psi(2,1)$$

Fermions

$$\psi(1,2) = -\psi(2,1)$$

Second quantization: work with creation operators at the given states

orthonormal base

lowering operator

$$\hat{\psi} = \sum_i \varphi_i(x)\hat{a}_i$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\hat{a}|\rangle = \sqrt{n}|n-1\rangle$$

Bosons:

$$[\hat{a}_i, \hat{a}_k^\dagger] = \delta_{ik}$$

$$[\hat{\psi}(x), \hat{\psi}(x')^\dagger] = \delta(x-x')$$

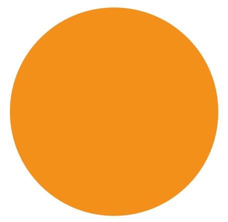
Fermions:

$$\{\hat{a}_i, \hat{a}_k^\dagger\} = \delta_{ik}$$

$$\{\hat{\psi}(x), \hat{\psi}(x')^\dagger\} = \delta(x-x')$$

Number states of bosons and fermions

Single orbit



Bosons

$$\hat{a}^\dagger |0\rangle = |1\rangle$$

$$\hat{a}^\dagger |1\rangle = \sqrt{2}\,|2\rangle$$

$$\vdots$$

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle$$

Hilbert space infinite on each site

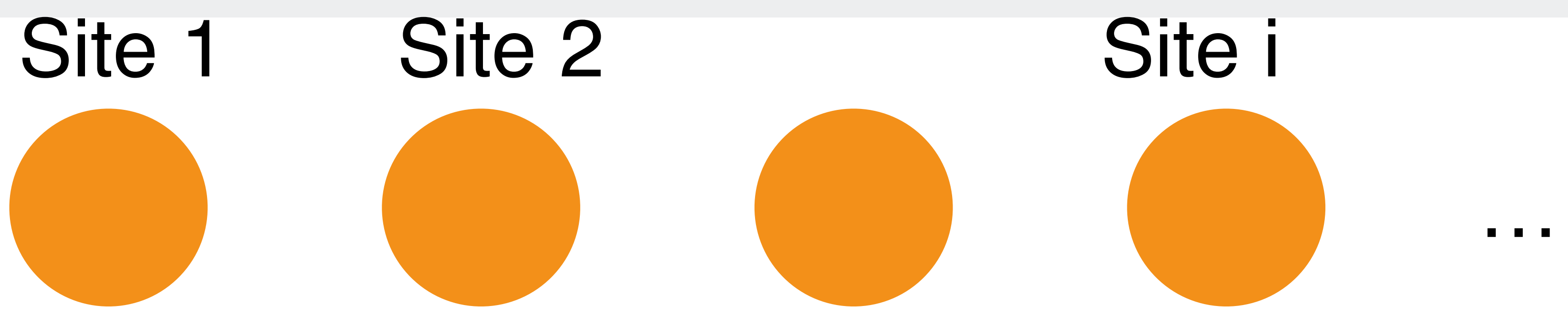
Fermions

$$\hat{a}^\dagger |0\rangle = |1\rangle$$

$$\hat{a}^\dagger |1\rangle = 0$$

Only two states (like for qubits)

Number states of bosons and fermions



<u>Bosons</u>

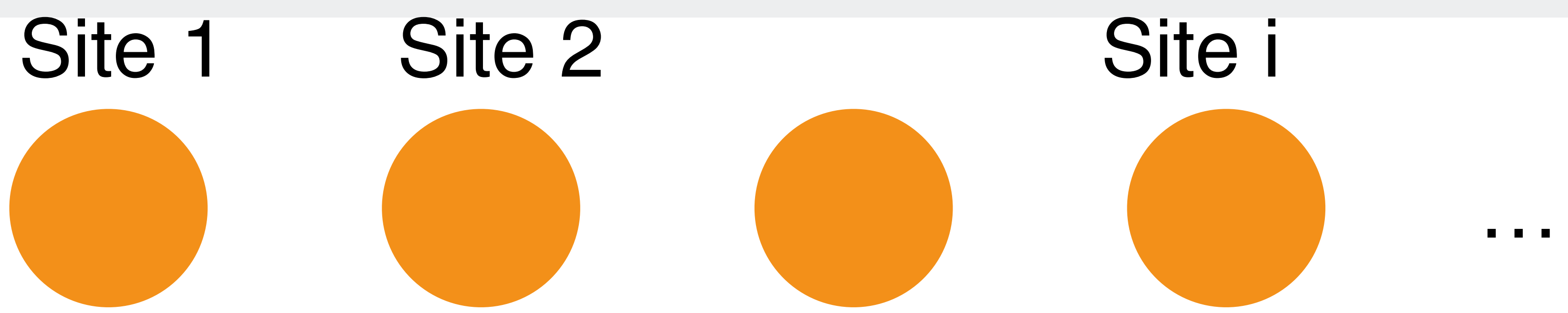
$$\hat{a}_j^\dagger |n_0, \cdots, n_j, \cdots\rangle = \sqrt{n_j + 1}\, |n_0, \cdots, n_j + 1, \cdots\rangle$$

<u>Fermions</u>

$$\hat{a}_j^\dagger |n_0, \cdots, n_j, \cdots\rangle = (-1)^{\sum_{i<j} n_i} \sqrt{1 - n_j}\, |n_0, \cdots, n_j + 1, \cdots\rangle$$

Sign problem

Number states of bosons and fermions



Bosons

$$\hat{a}_j^\dagger |n_0, \cdots, n_j, \cdots\rangle = \sqrt{n_j + 1}\,|n_0, \cdots, n_j + 1, \cdots\rangle$$

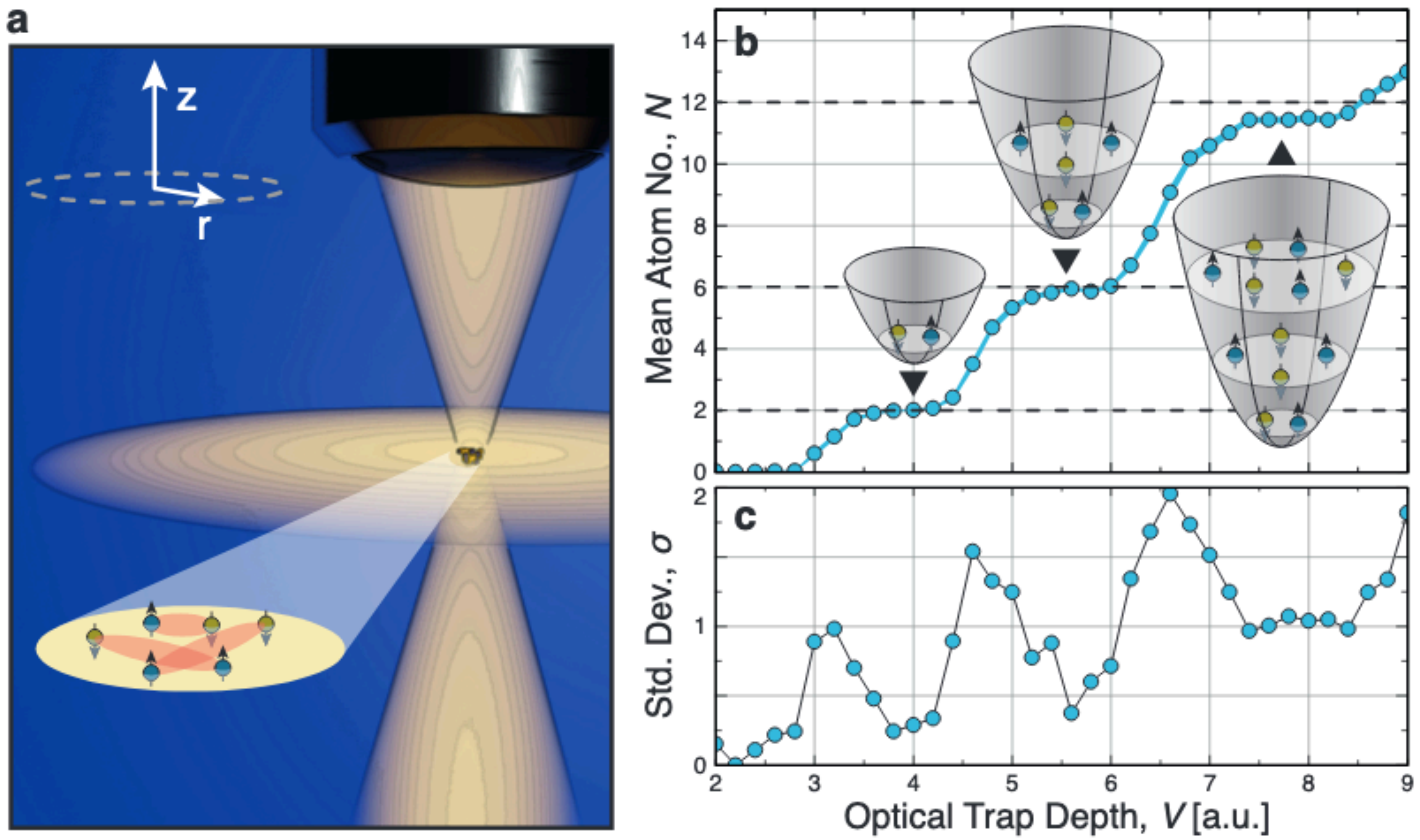
Fermions

$$\hat{a}_j^\dagger |n_0, \cdots, n_j, \cdots\rangle = (-1)^{\sum_{i<j} n_i}\sqrt{1 - n_j}\,|n_0, \cdots, n_j + 1, \cdots\rangle$$

- Quite complicated even on quantum computers
- Solve this in natural implementations ?

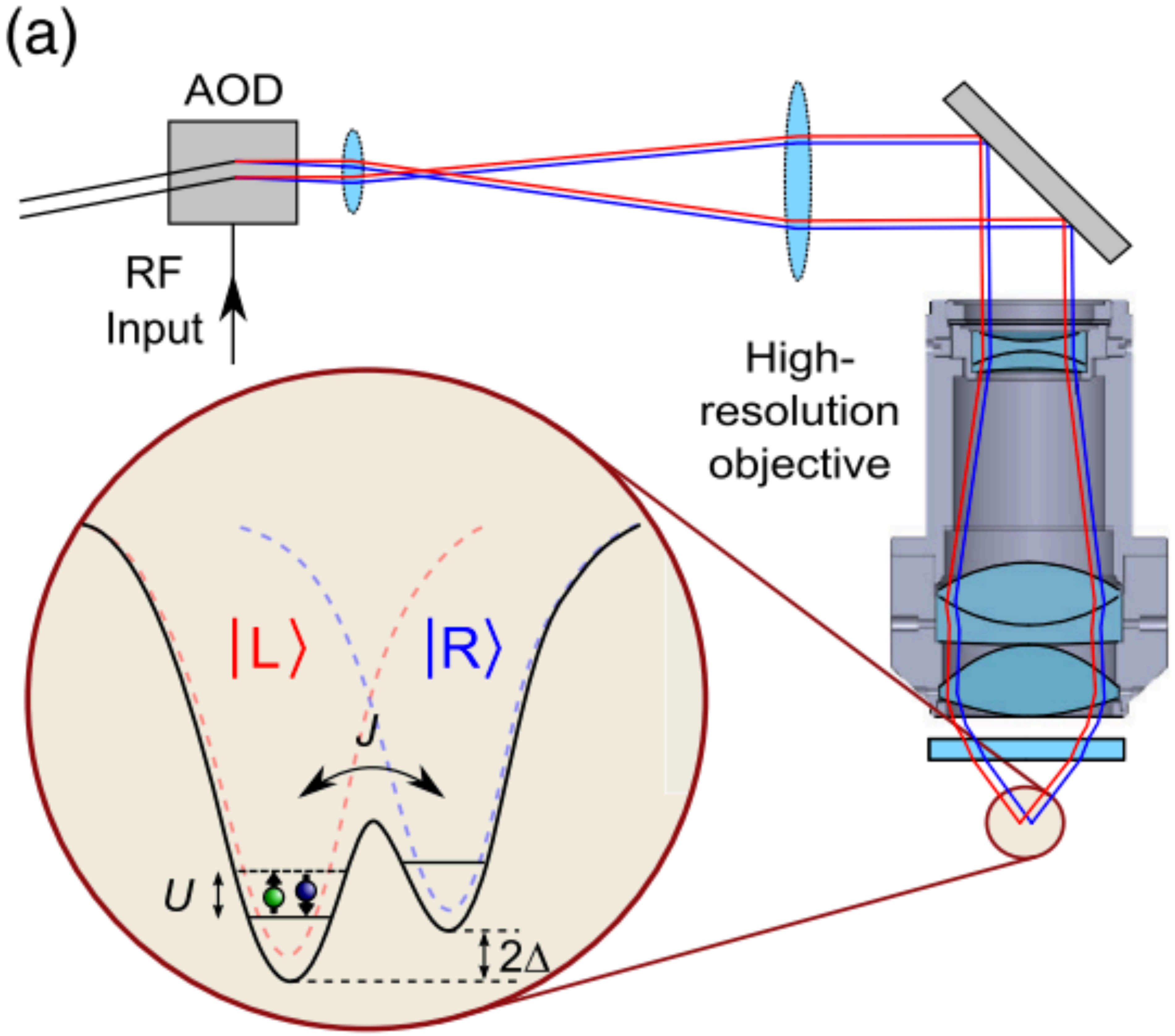
Sign problem

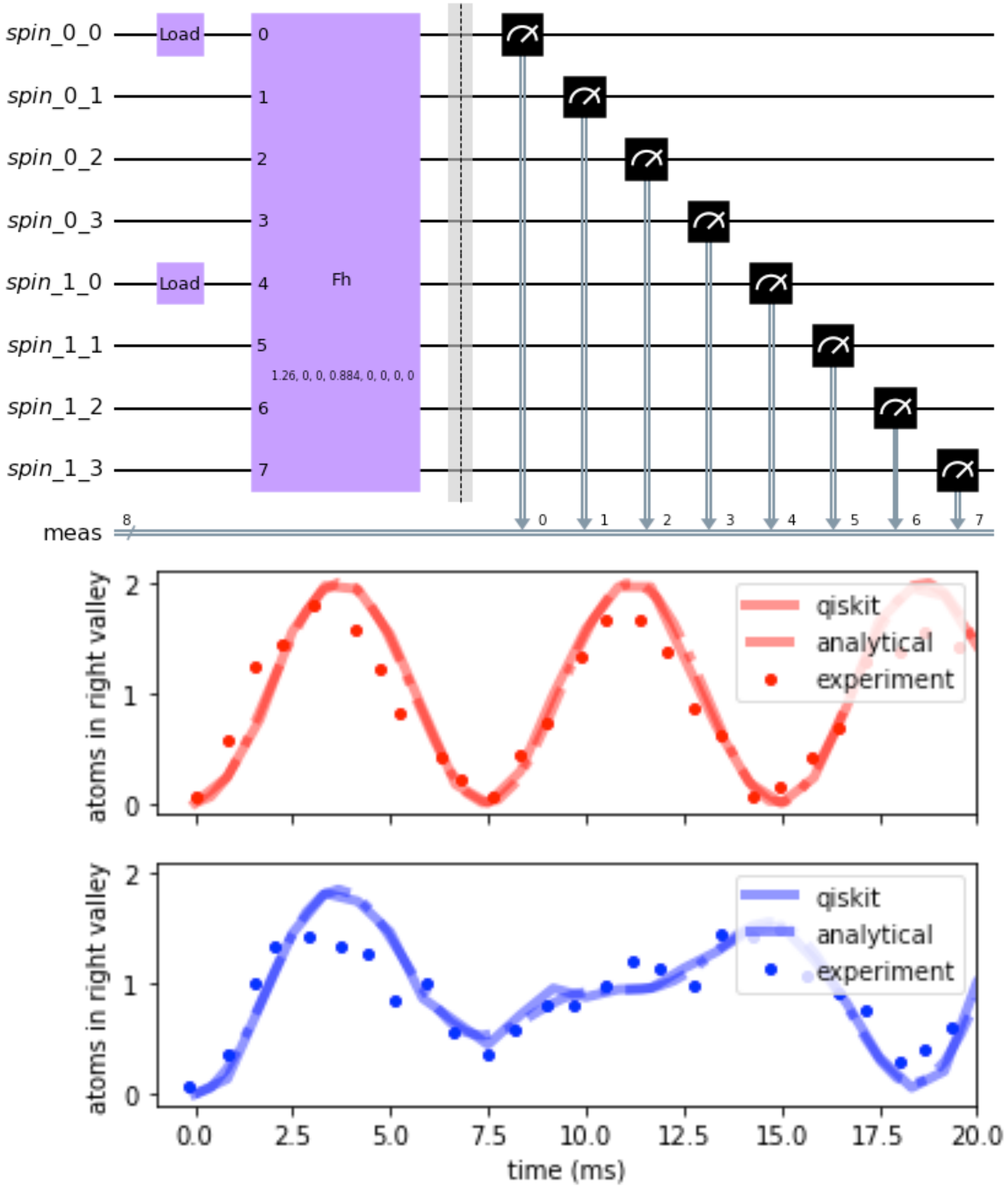
One fermion at a time



L. Bayha et al., arXiv 2004.14761 (2020).

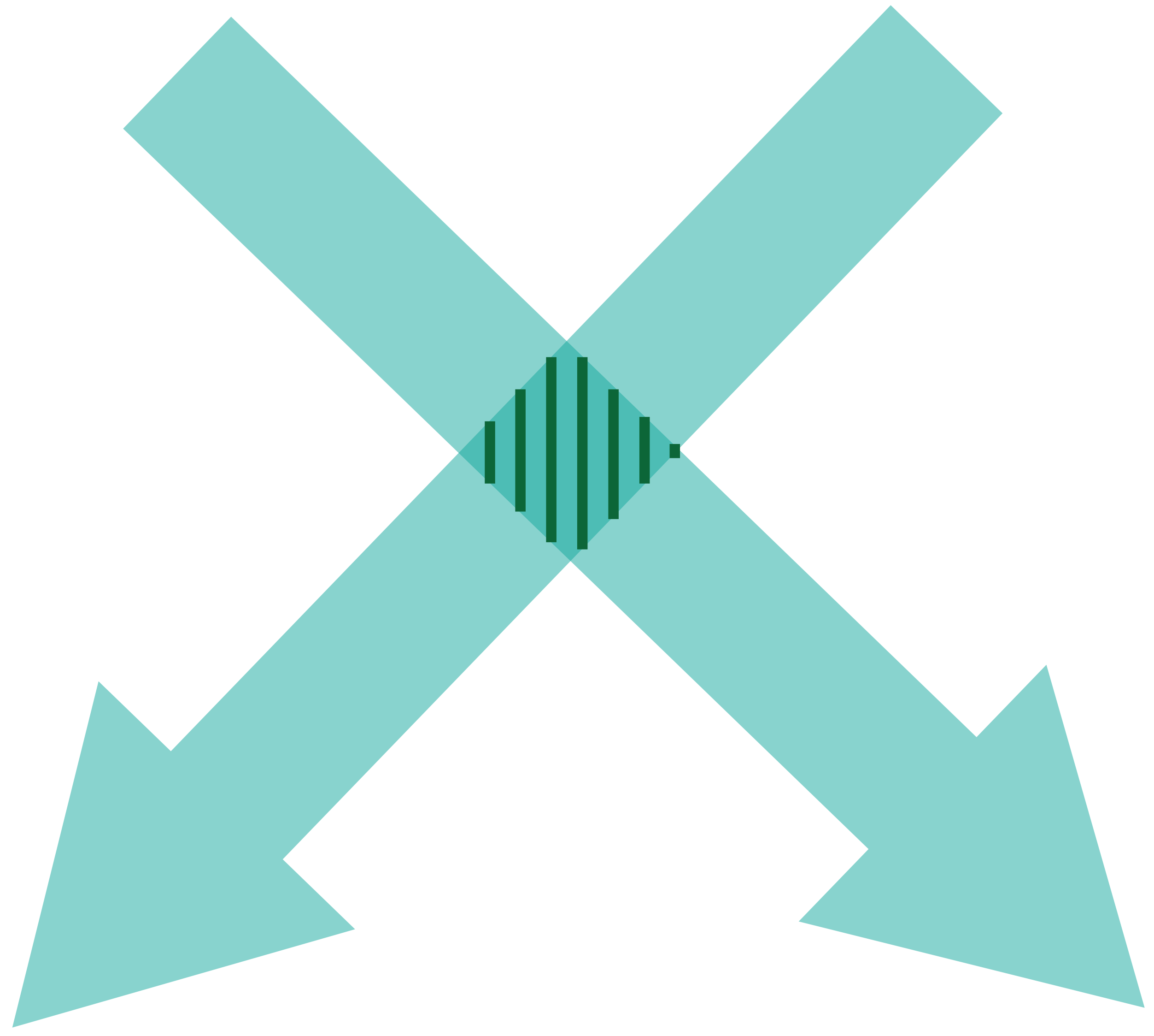
Fermions in different tweezers

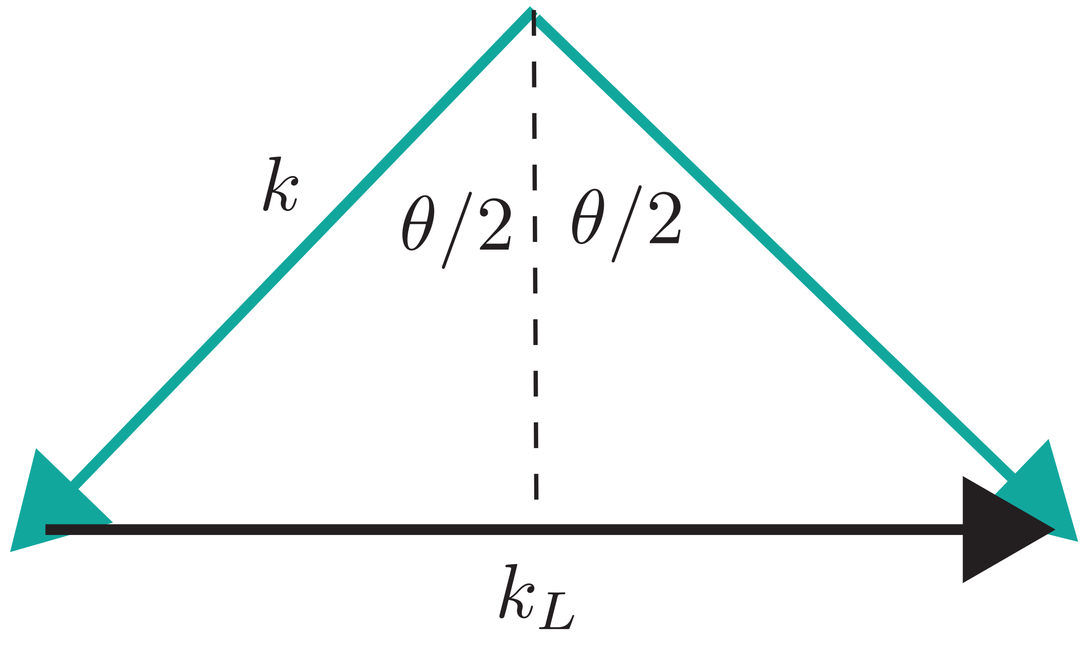




S. Murmann. Physical Review Letters, 114, 080402 (2015)

Now let's go real big - the optical lattice





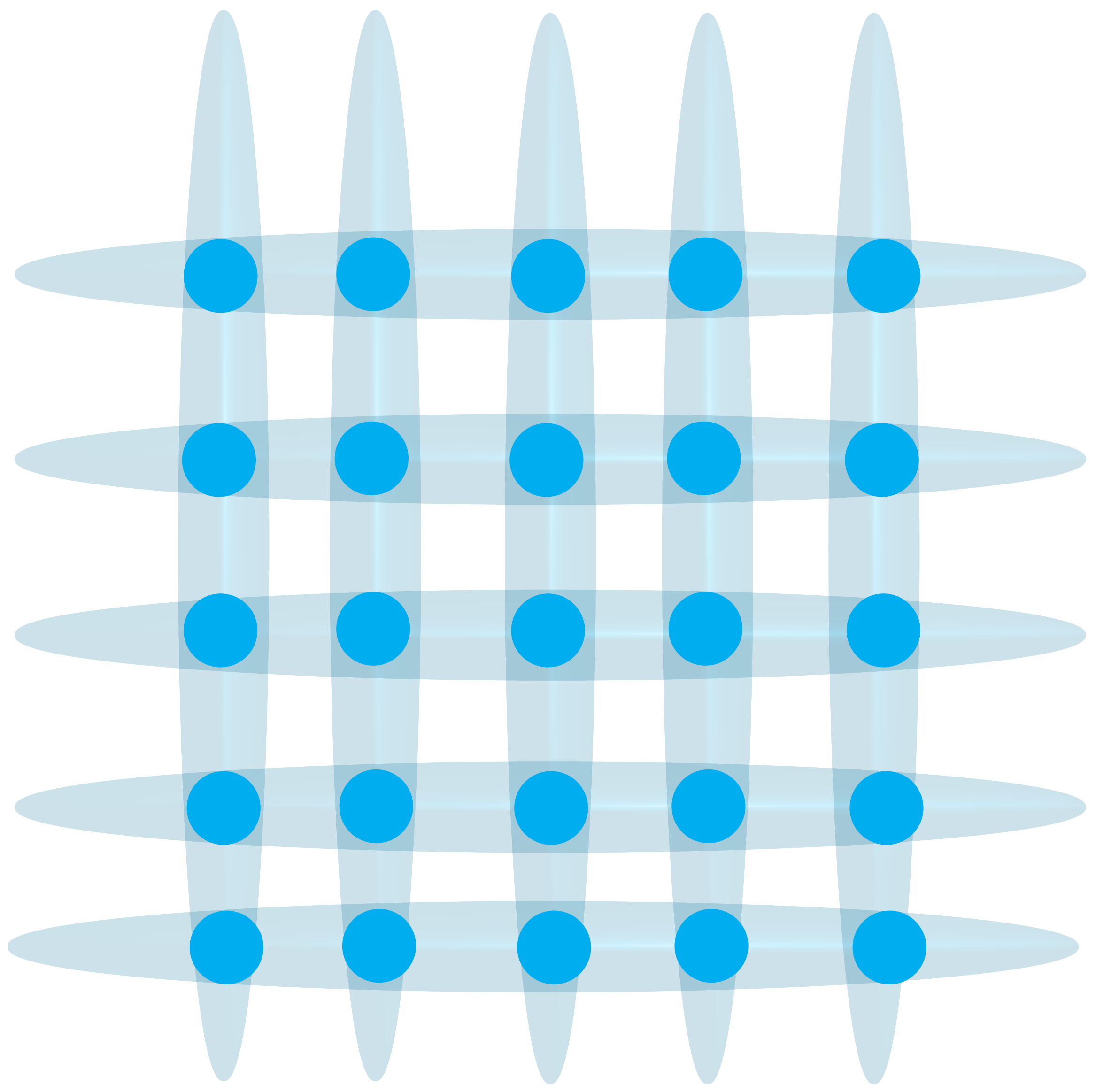
$$\sin(\theta/2) = \frac{k_L/2}{k}$$

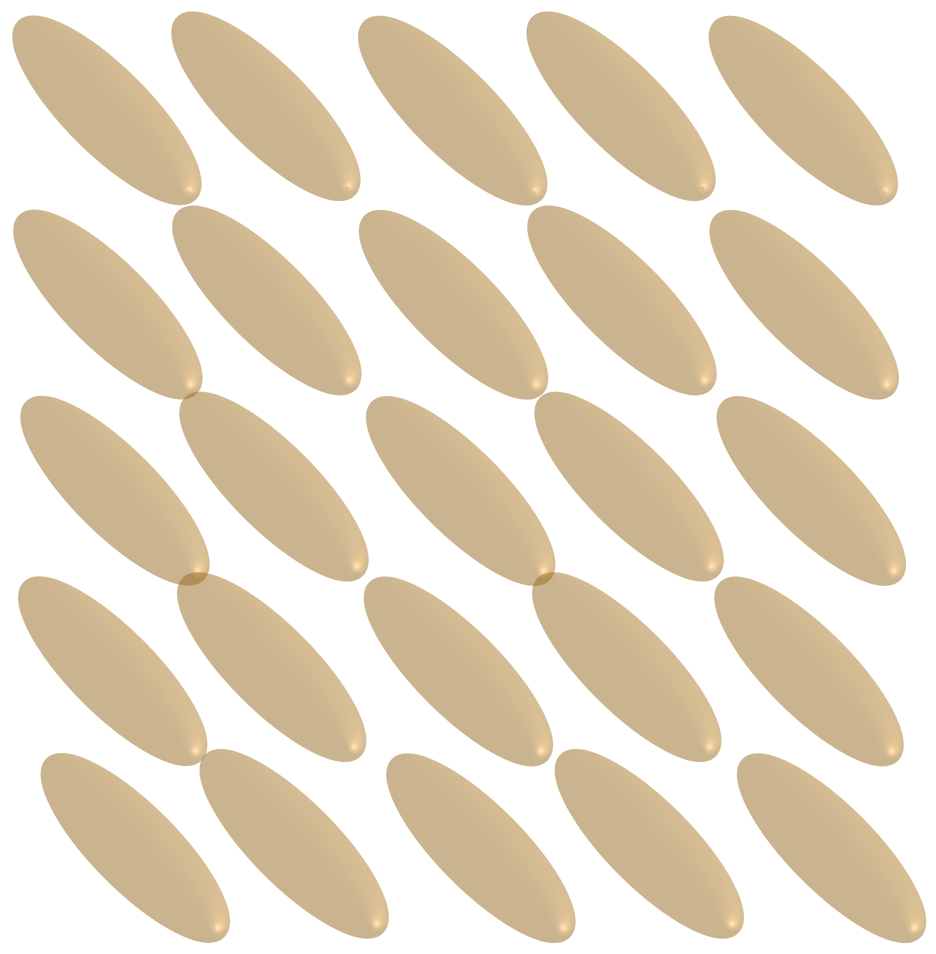
Lattice spacing: $$a_L = \frac{\lambda}{2\sin(\theta/2)}$$

Optical lattice with potential: $$V_L = V_0 \sin^2(k_L x) \qquad k_L = \frac{\pi}{a_L}$$

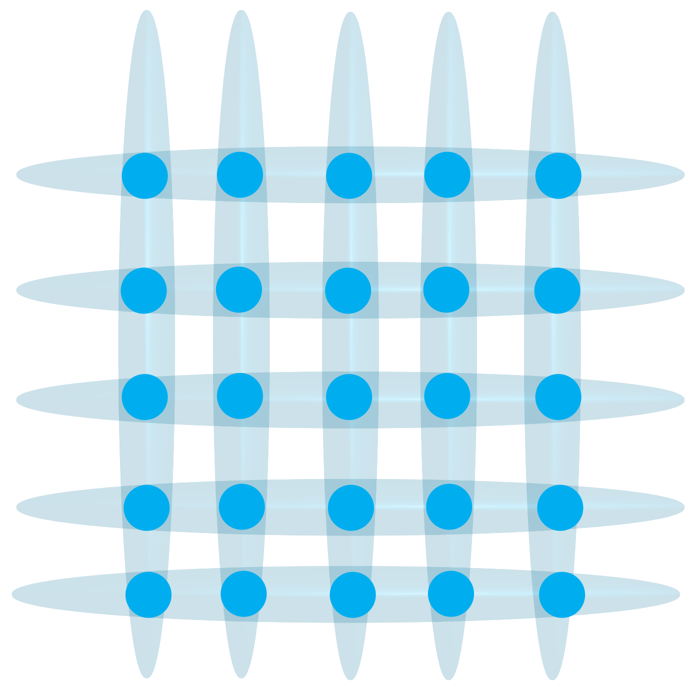
Higher dimensional lattice

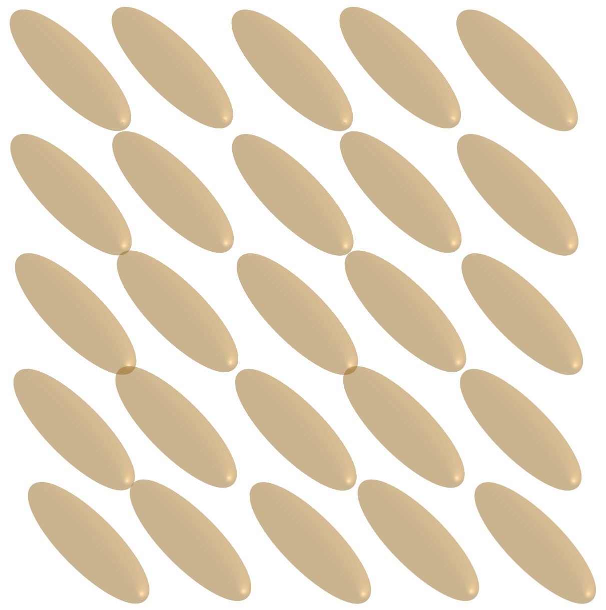
Cross the polarization !

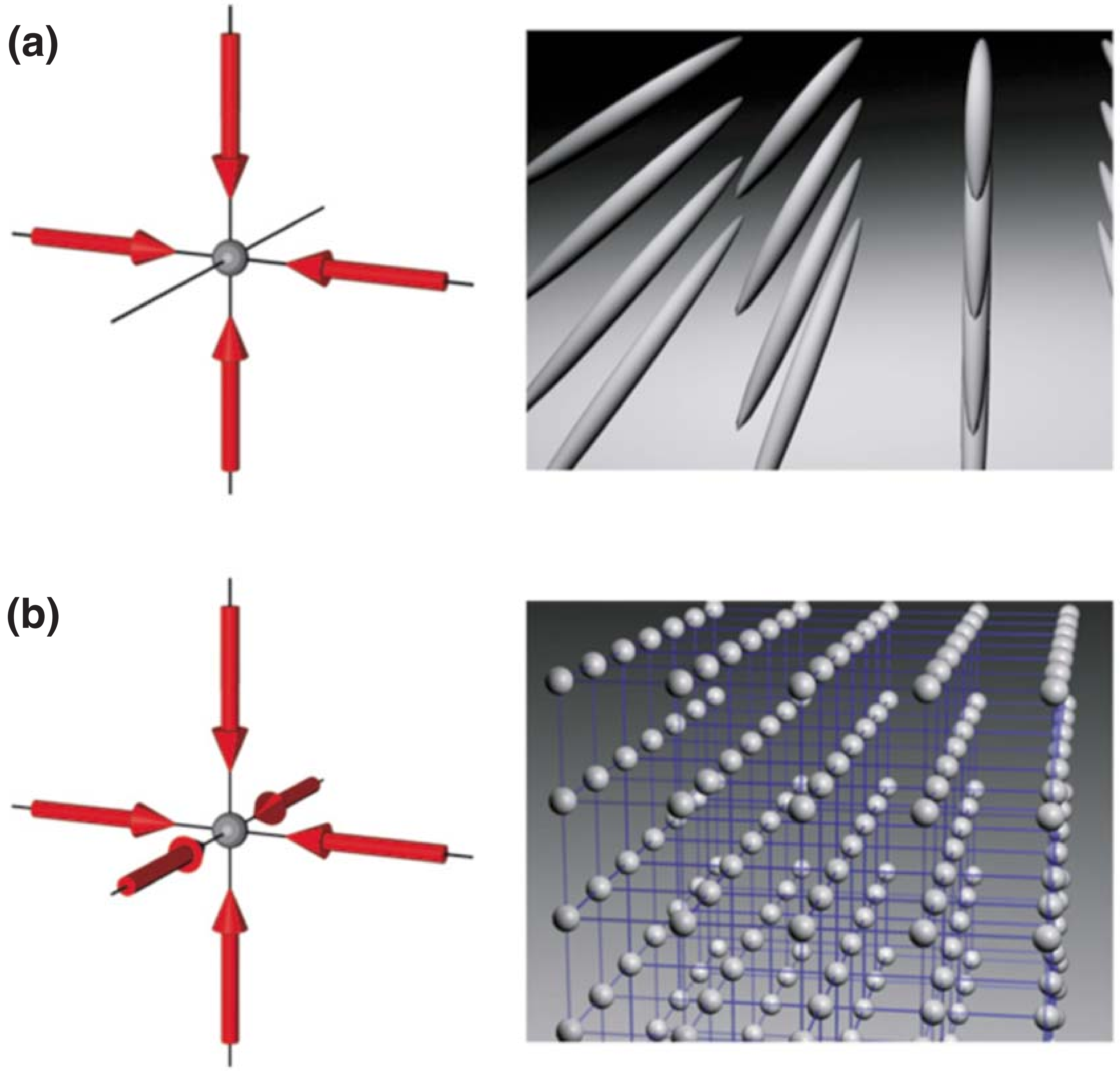




Higher dimensional lattice





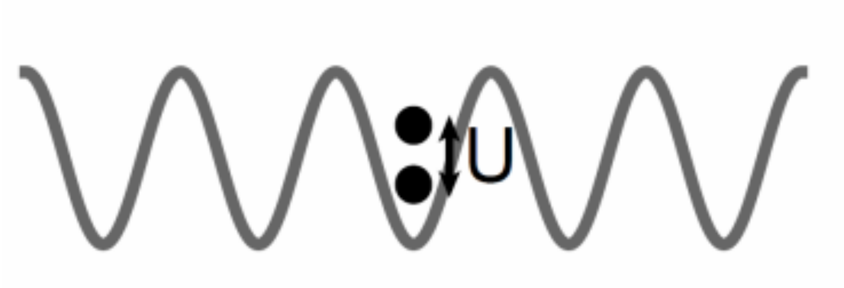


Zwerger et al. RMP

Bose-Hubbard Model

$$H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + \mathrm{h.c.}) + \frac{U}{2} \sum_i n_i (n_i - 1)$$

tunneling J

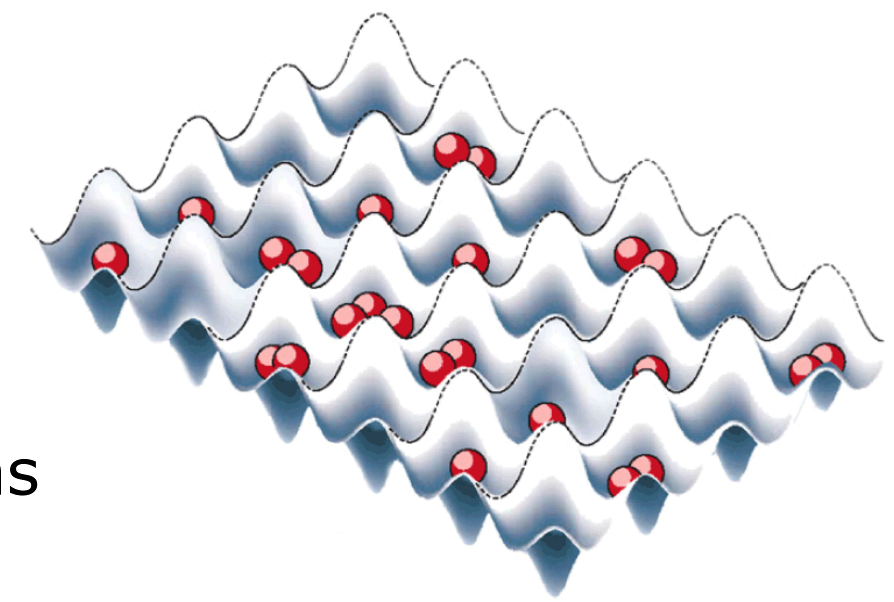


interaction U

$U \ll J$

Superfluid

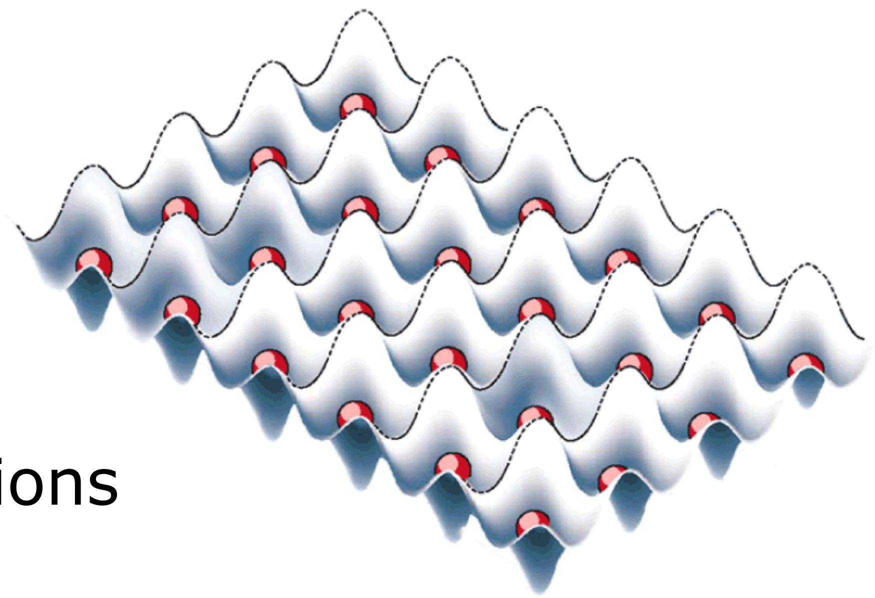
- Large number fluctuations
- Coherent state on-site

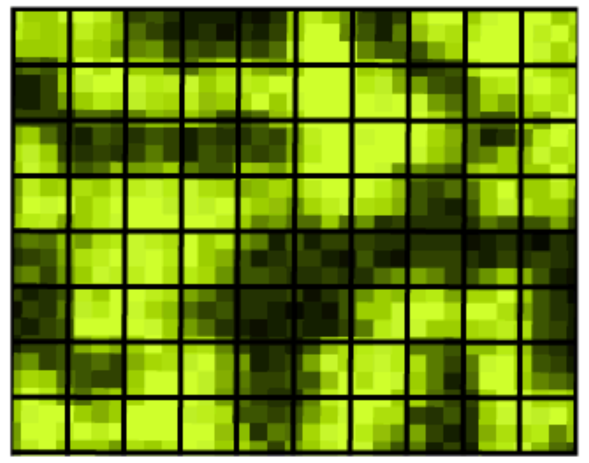


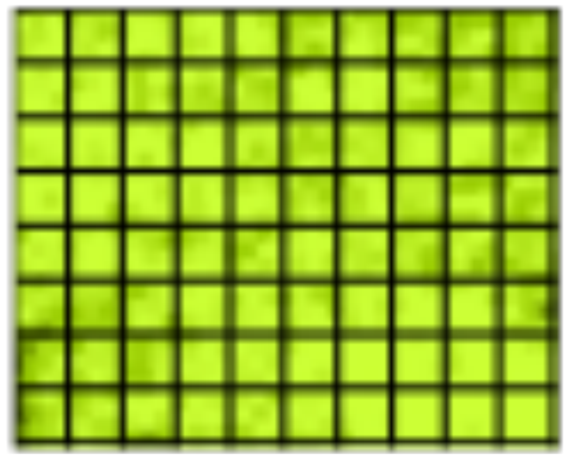
$J \ll U$

Mott insulator

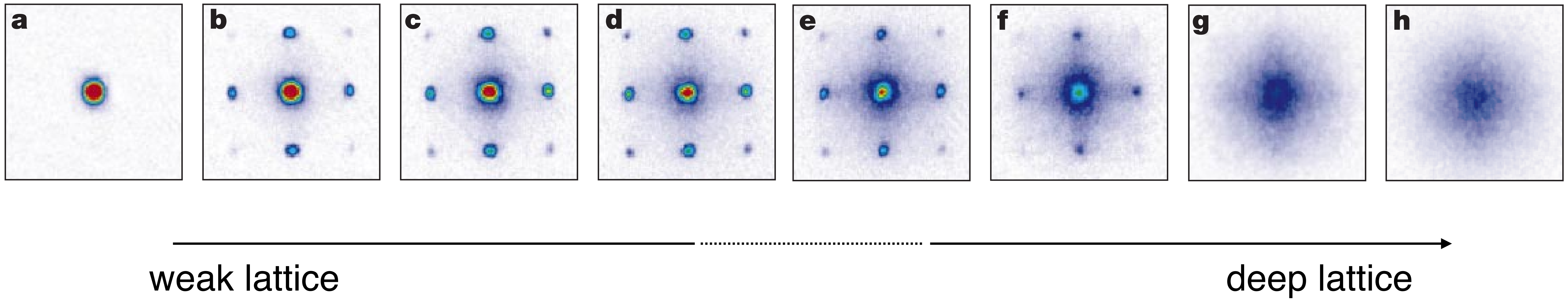
- No number fluctuations
- Fock state on-site

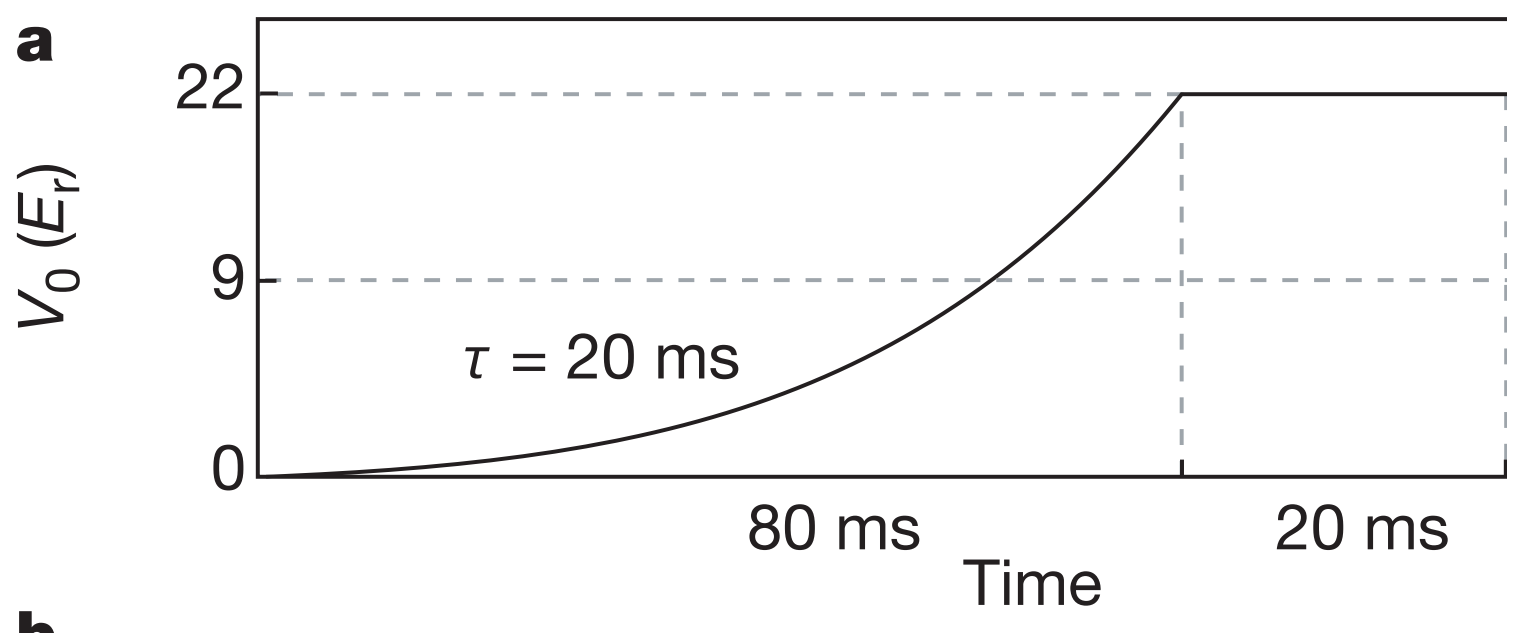






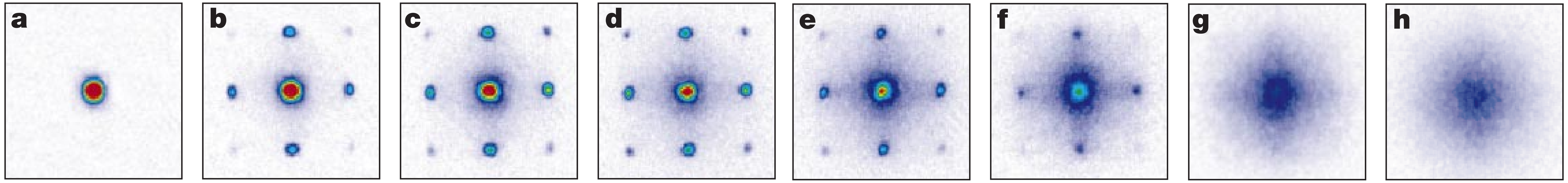
Observation



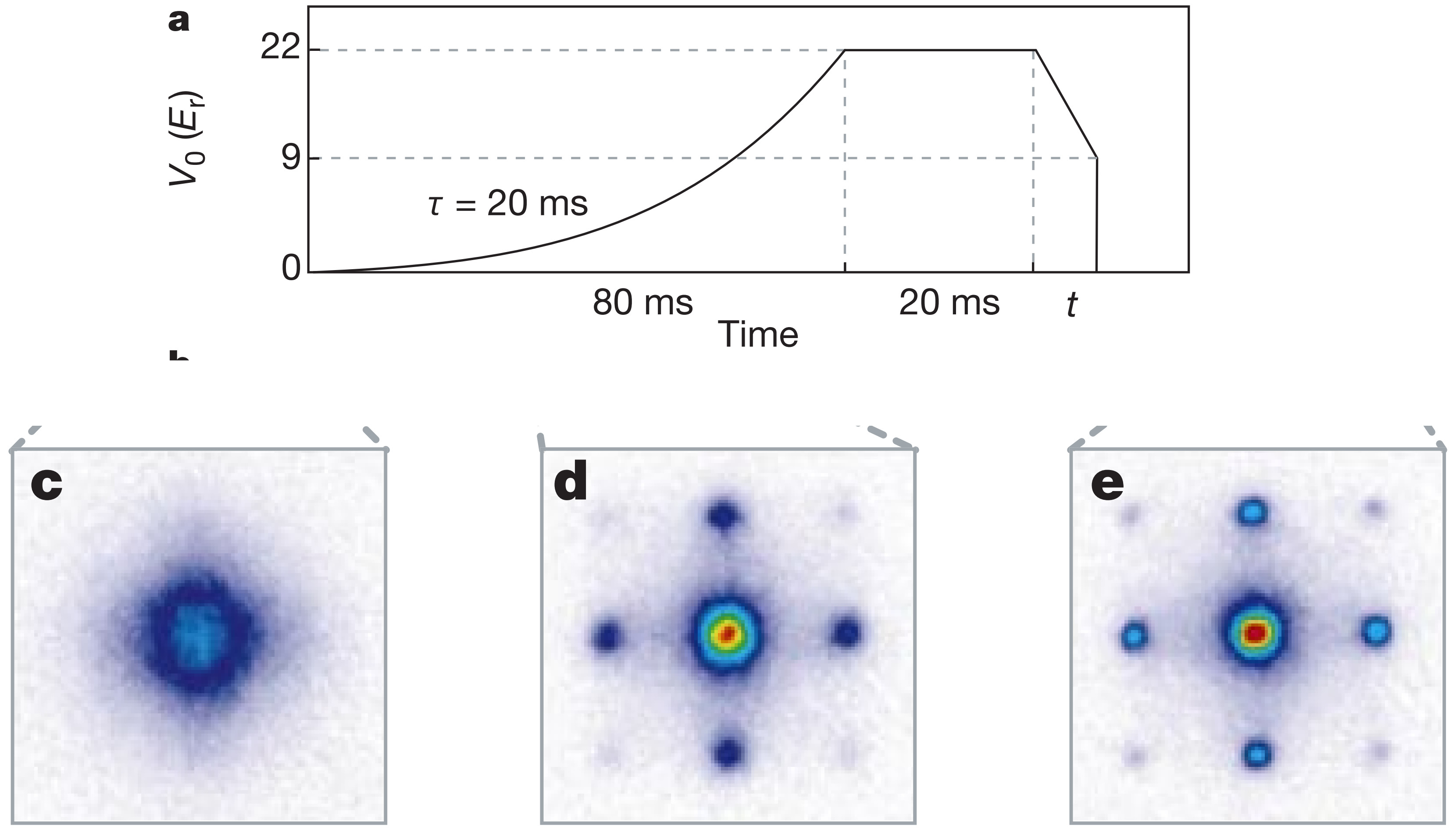


Greiner *et al., Nature* **415** 39 (2002)

Observation

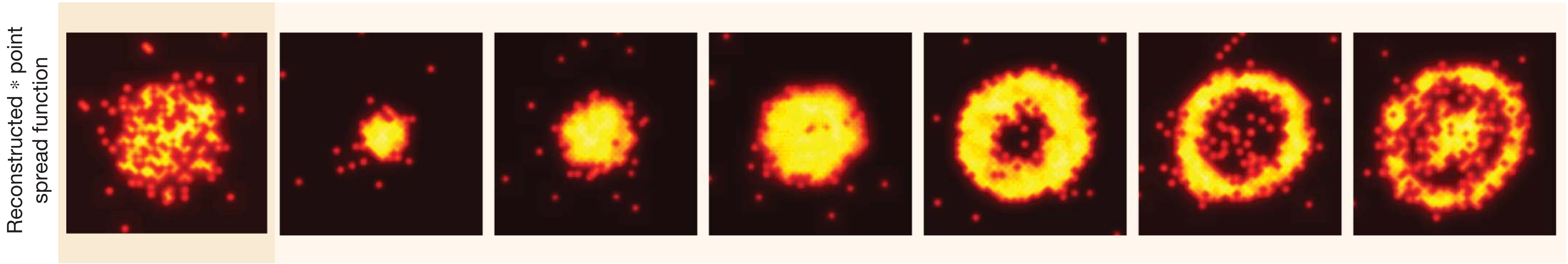


<u>And it is reversible:</u>

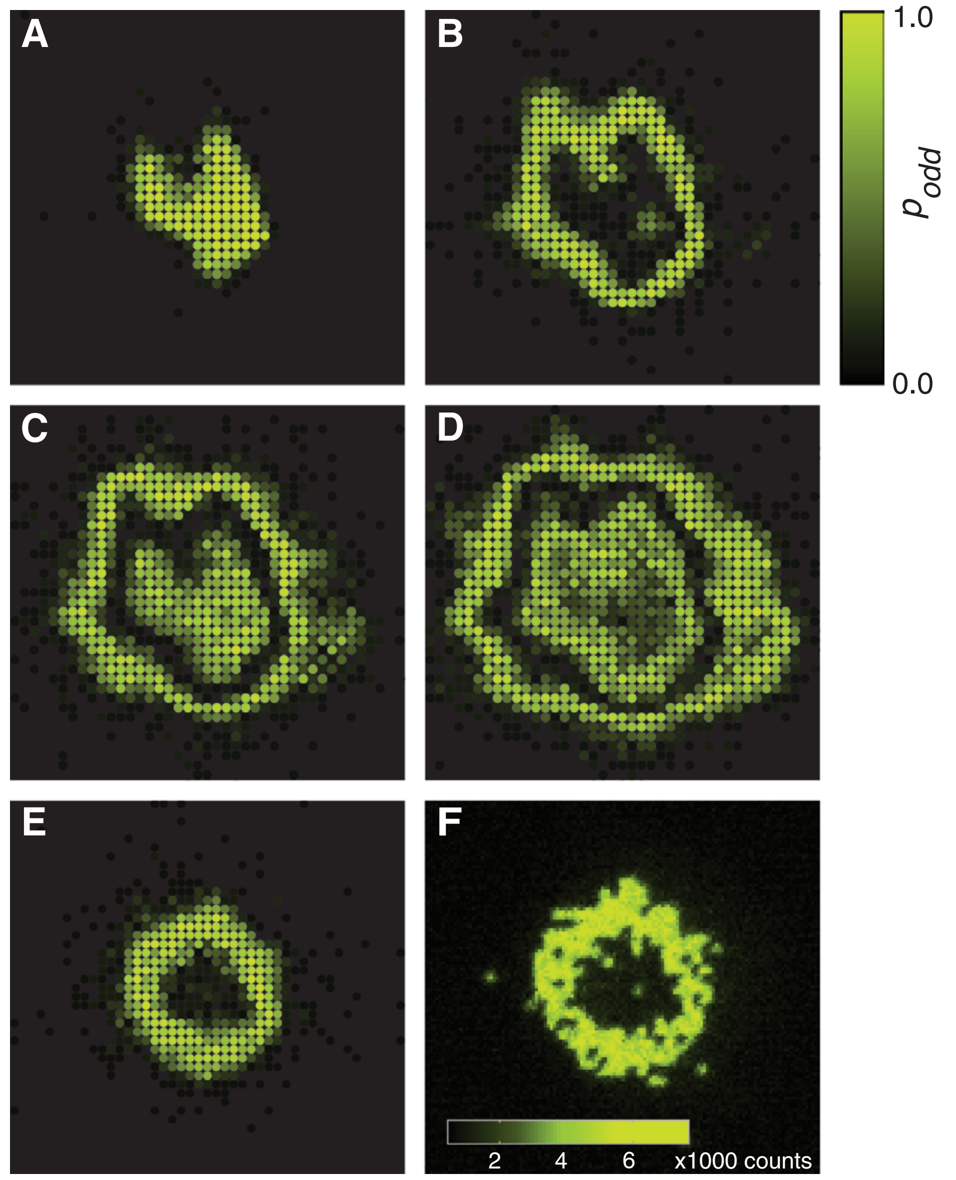


Greiner *et al., Nature* **415** 39 (2002)

In-situ observation



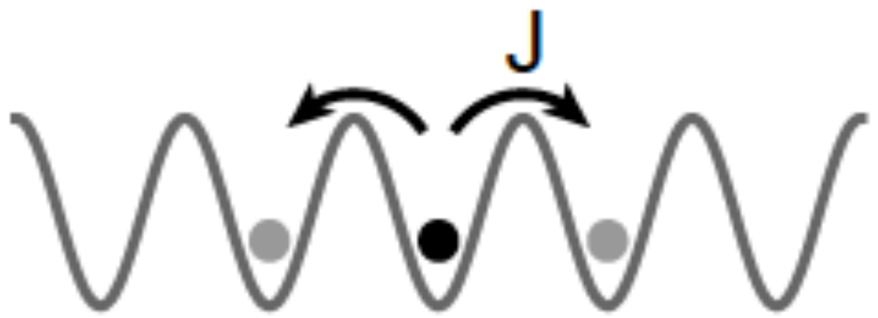
J. Sherson *et al.* *Nature* **467** 68 (2010)

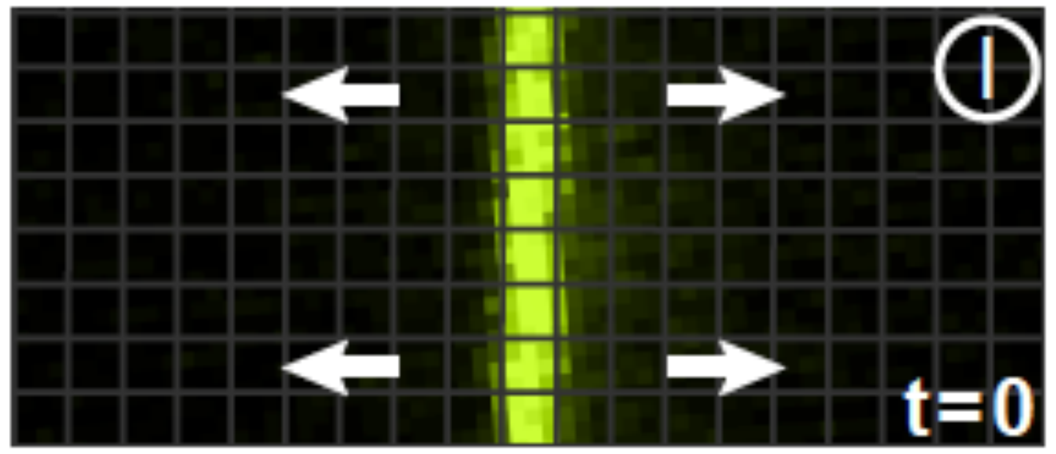


W. Bakr *et al.* *Science* **329** 547 (2010)

Single-Particle Quantum Walk

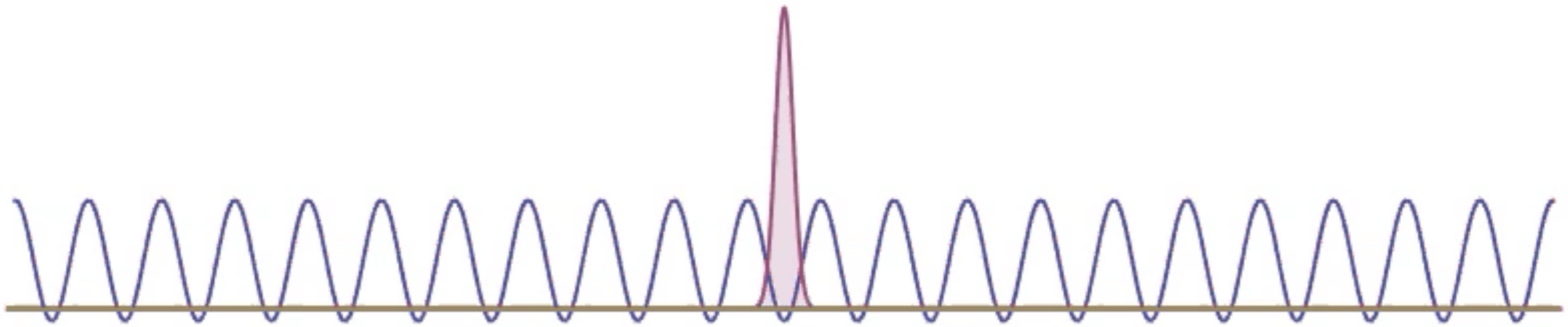
Single realization

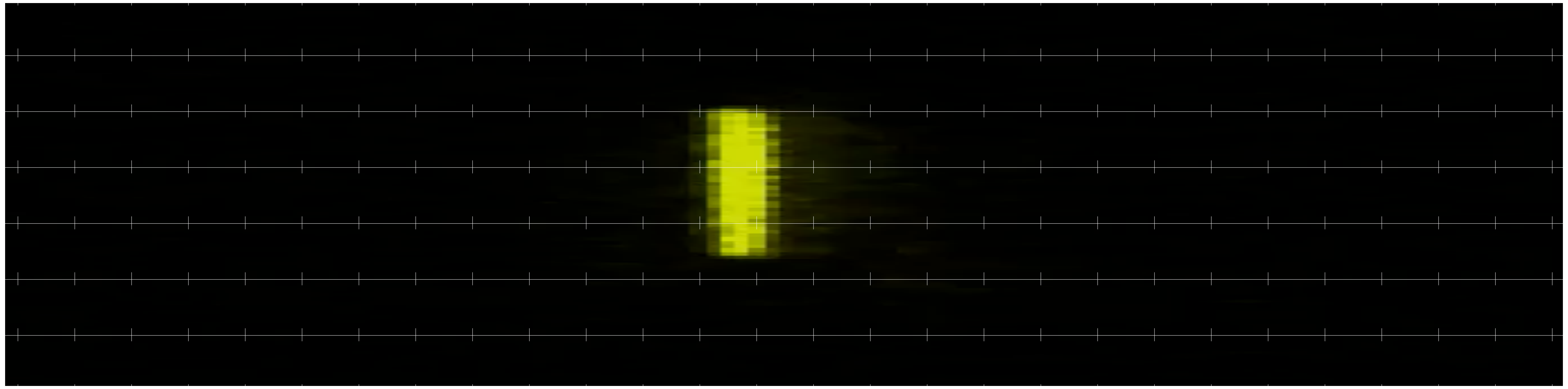




Thanks to Philipp Preiss for the slides

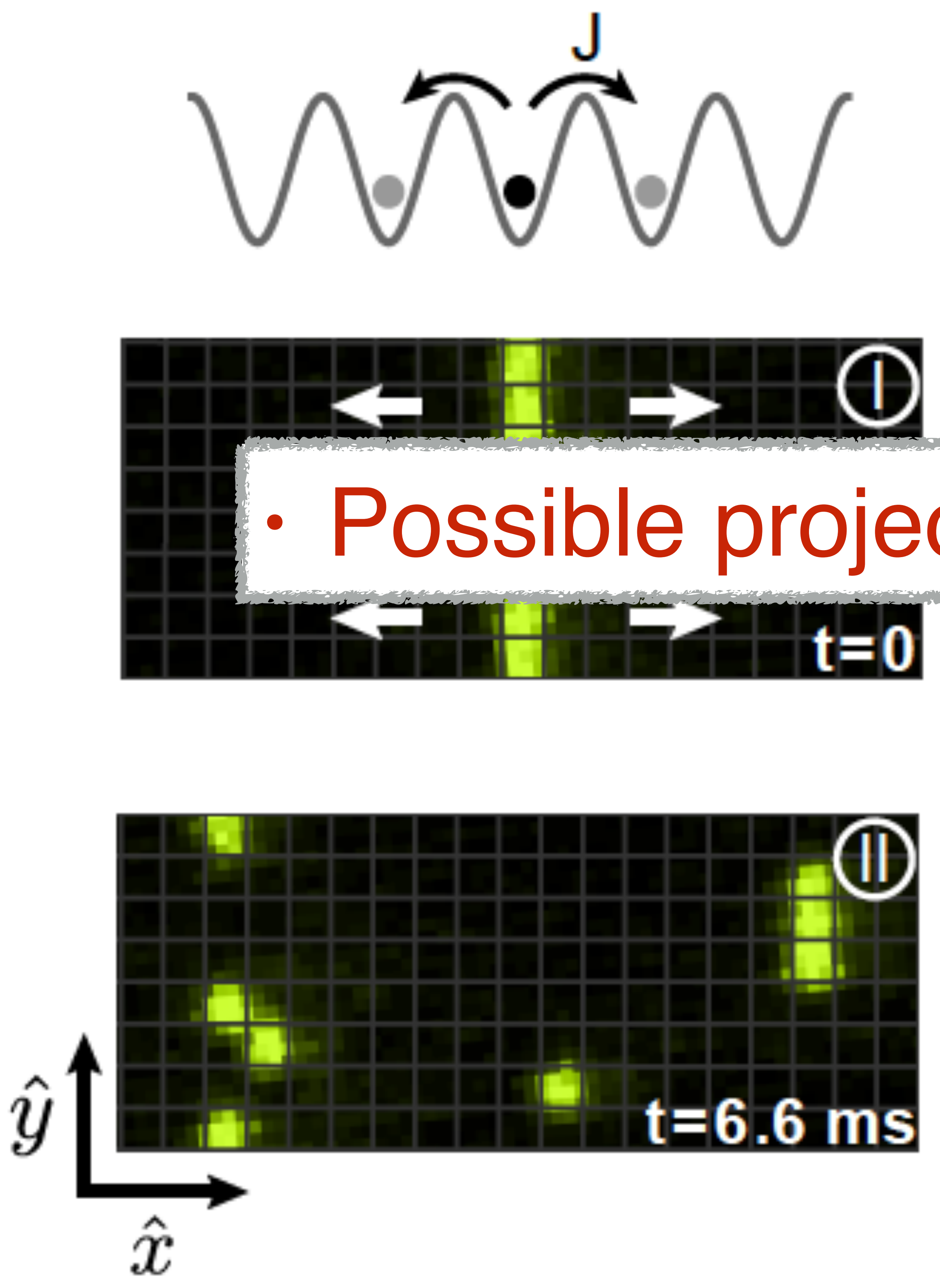
Single-Particle Quantum Walk



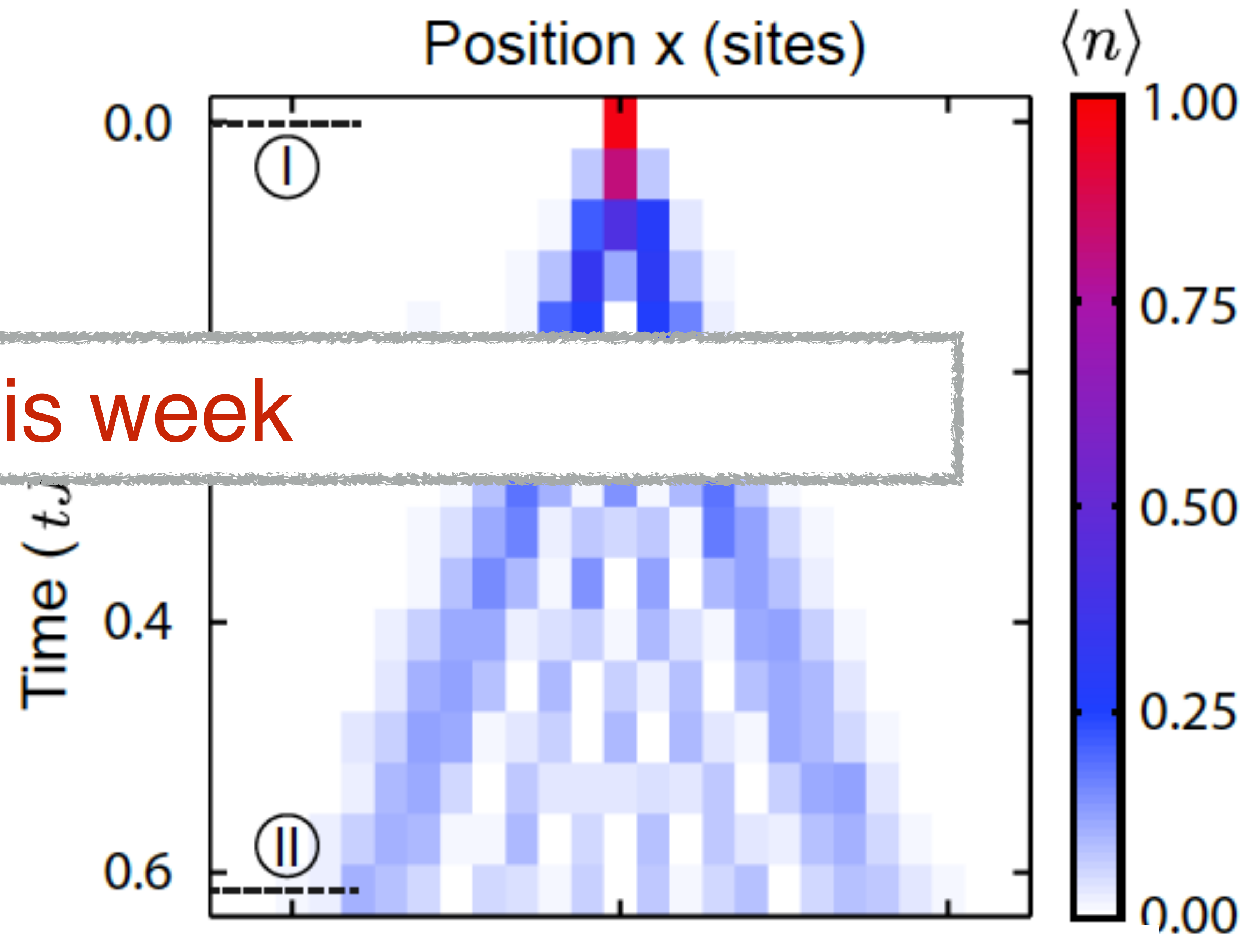


Single-Particle Quantum Walk

Single realization



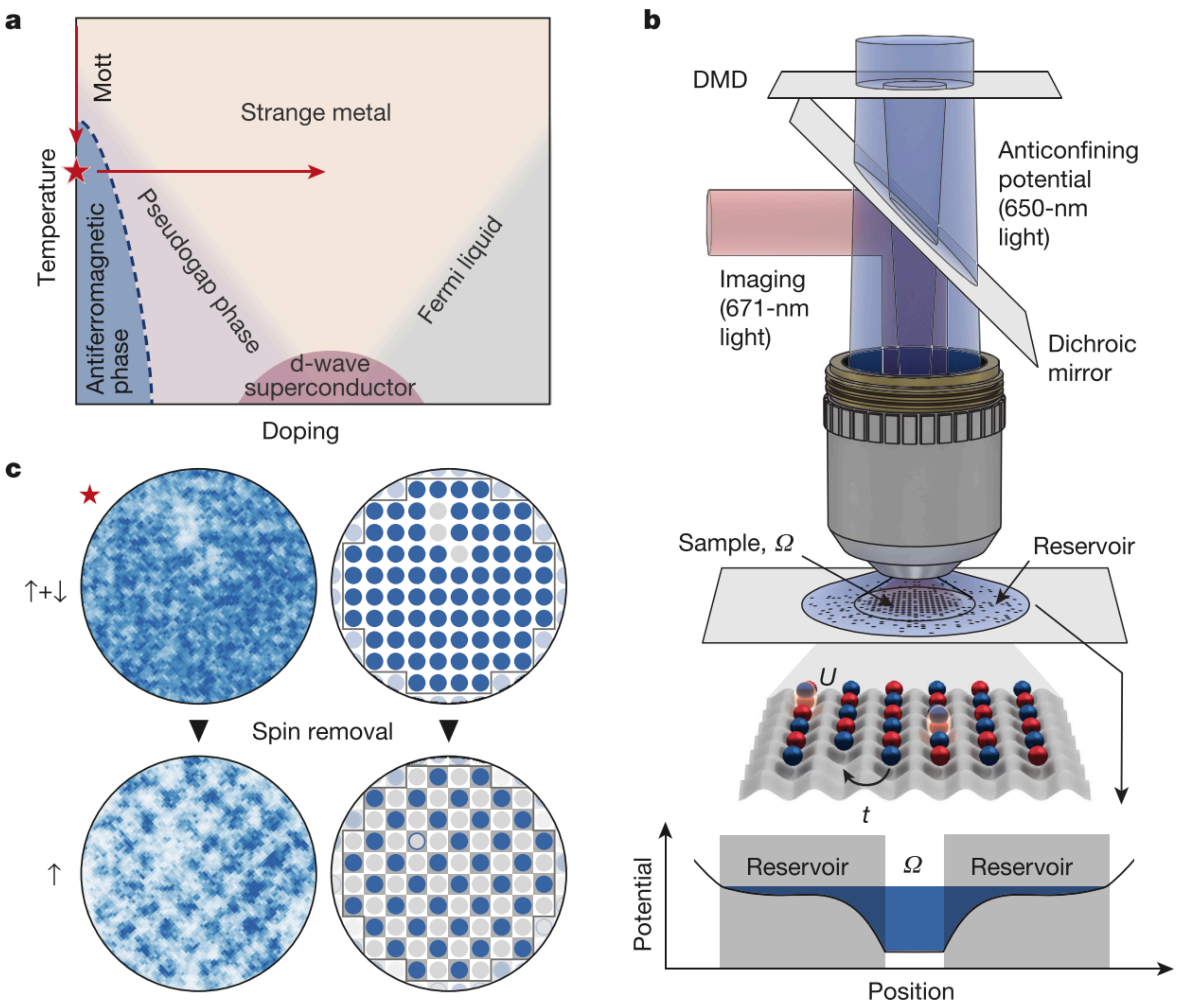
Average density evolution



- Possible project for this week

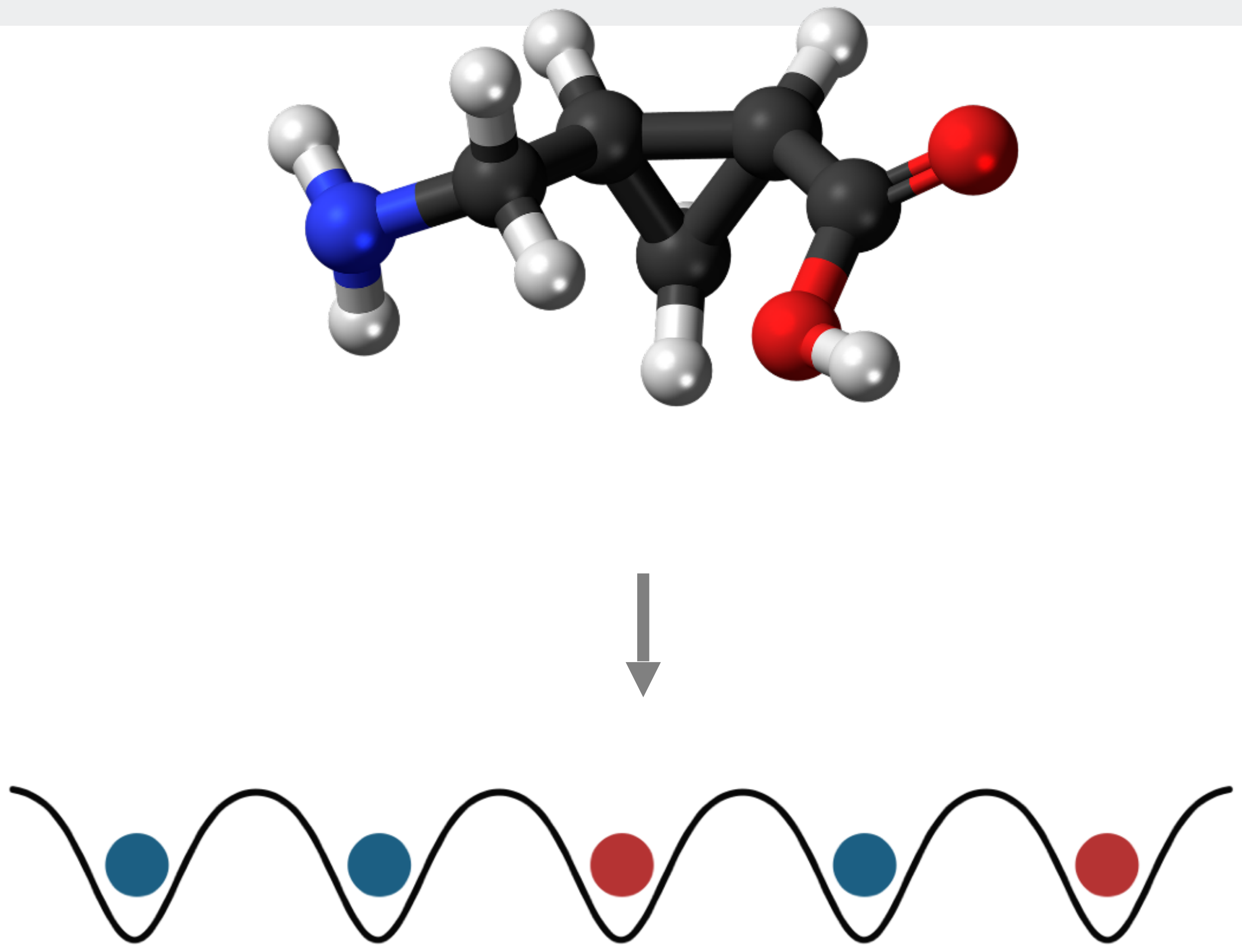
P. M. Preiss *et al.*, Science **347** 1229 (2015)

The Fermi-Hubbard model

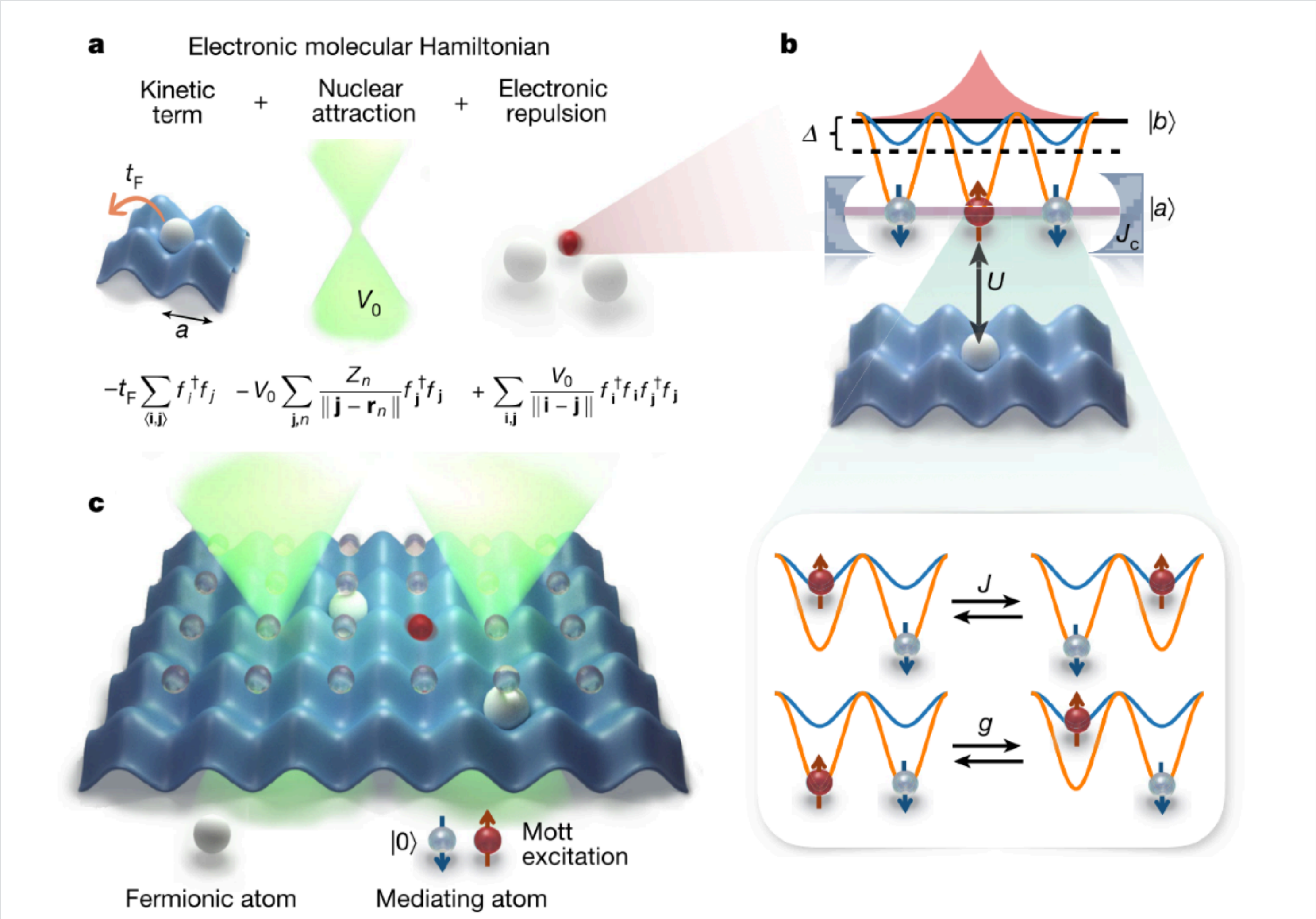


A. Mazurenko et al., Nature 545, 462 (2016).

Putting chemistry into the machines - electronic structure

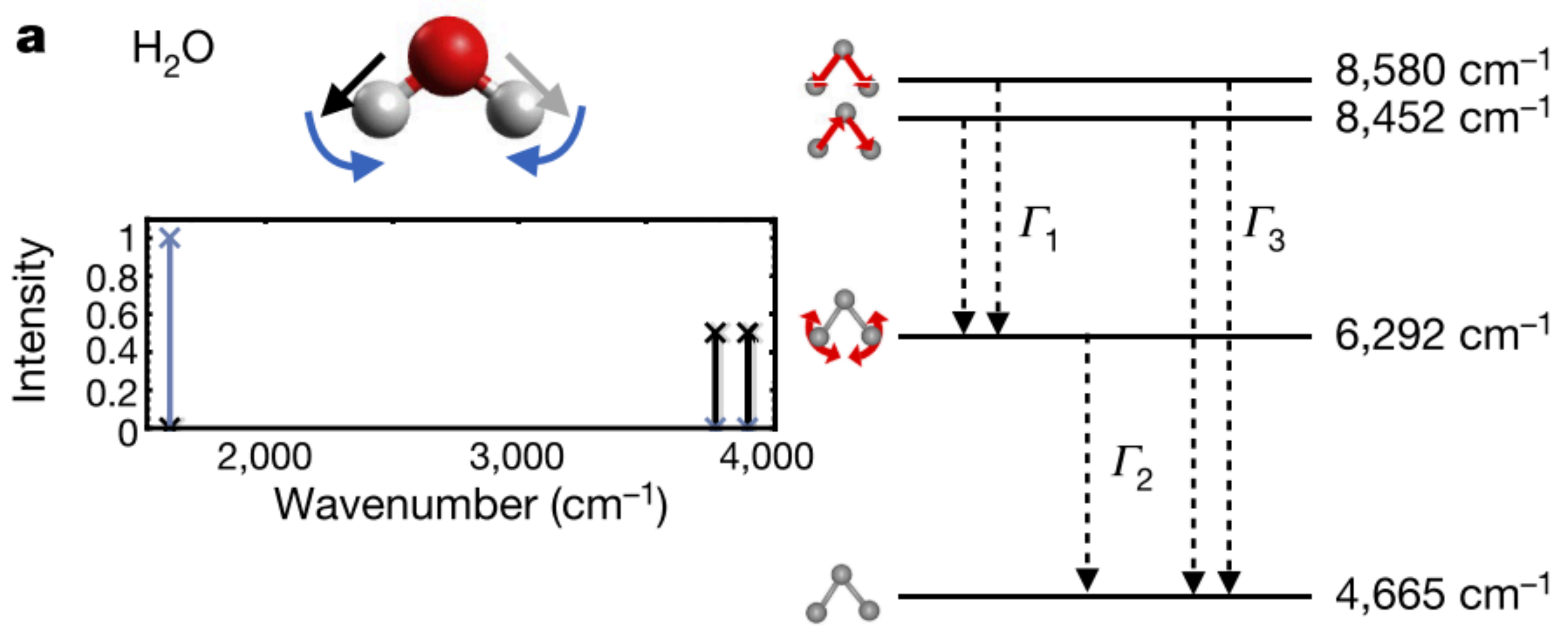


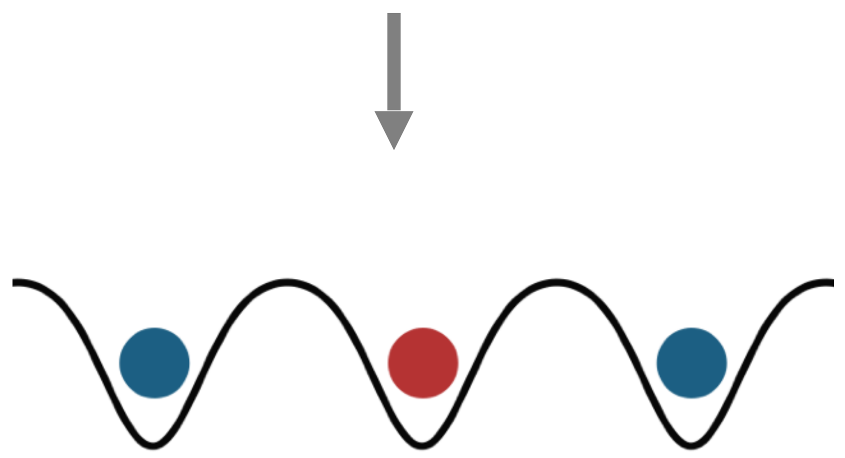
What are the electronic properties of molecules ?



J. Argüello-Luengo et al.
Nature 574, 215 (2019).

Putting chemistry into the machines - vibrational spectra





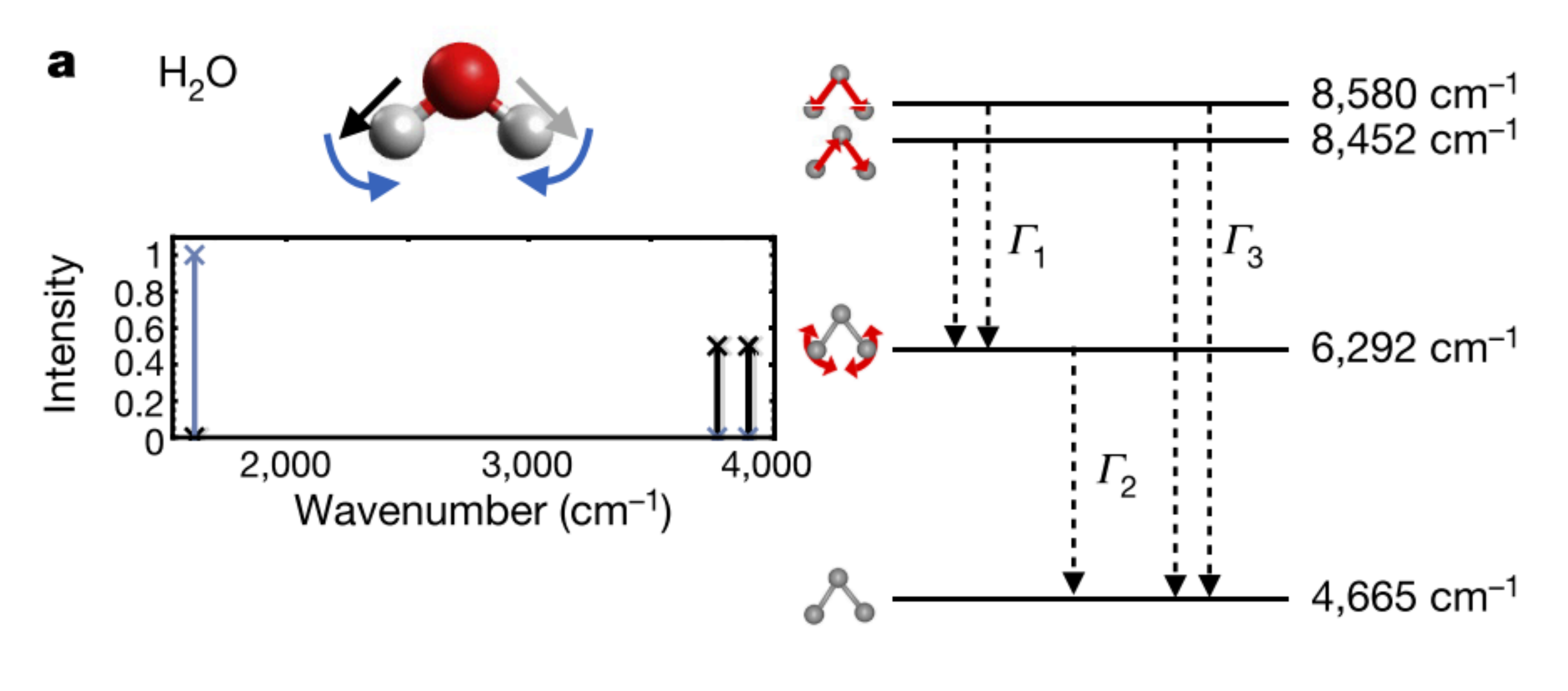
$$\hat{H}_a = \hat{H} + \hbar \sum_{i \leq j} \frac{x_{ij}}{2} \sqrt{\omega_i \omega_j} (a_i^\dagger a_i + a_j^\dagger a_j + 2 a_i^\dagger a_j^\dagger a_i a_j)$$

$$\hat{H} = \sum_i \hbar \omega_i a_i^\dagger a_i$$

C. Sparrow, Nature 557, 660 (2018).

Putting chemistry into the machines

Vibrational spectra



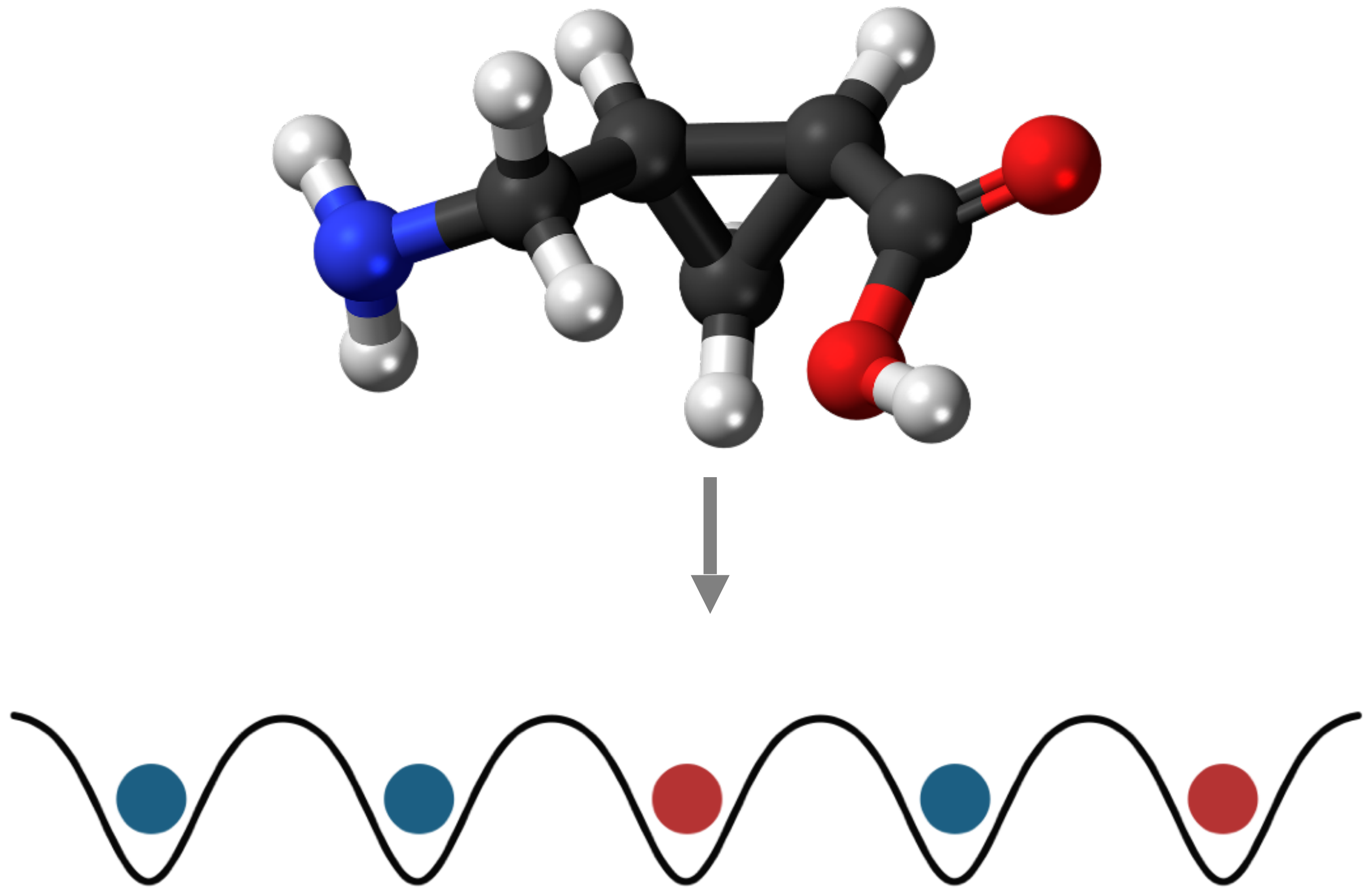
$$\hat{H} = \sum_i \hbar\omega_i a_i^\dagger a_i$$

$$\hat{H}_a = \hat{H} + \hbar \sum_{i \le j} \frac{x_{ij}}{2} \sqrt{\omega_i \omega_j} (a_i^\dagger a_i + a_j^\dagger a_j + 2 a_i^\dagger a_j^\dagger a_i a_j)$$

Looks an awful lot like **bosonic** Hubbard problem

C. Sparrow, Nature 557, 660 (2018).

Electron structure

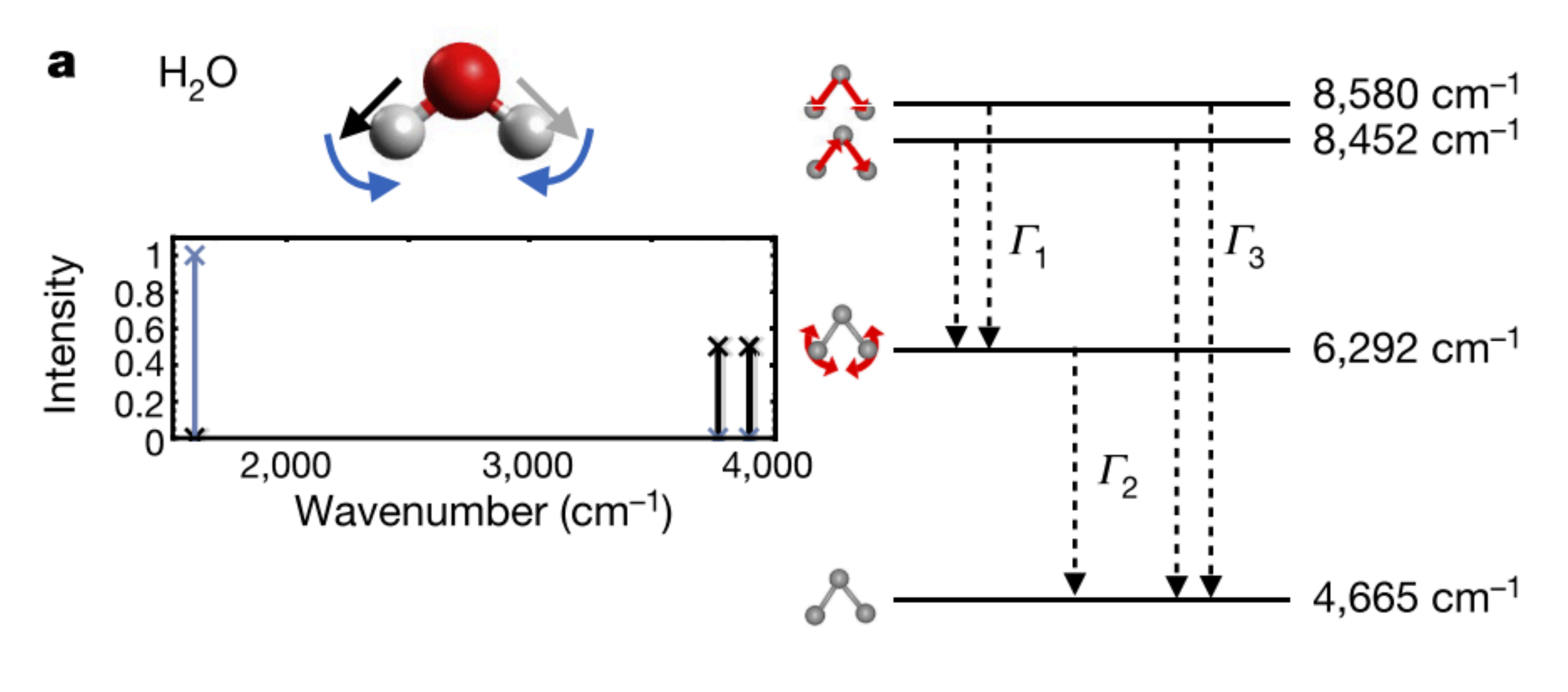


Looks an awful lot like **fermionic** Hubbard problem

J. Argüello-Luengo et al. Nature 574, 215 (2019).

Putting chemistry into the machines

Vibrational spectra



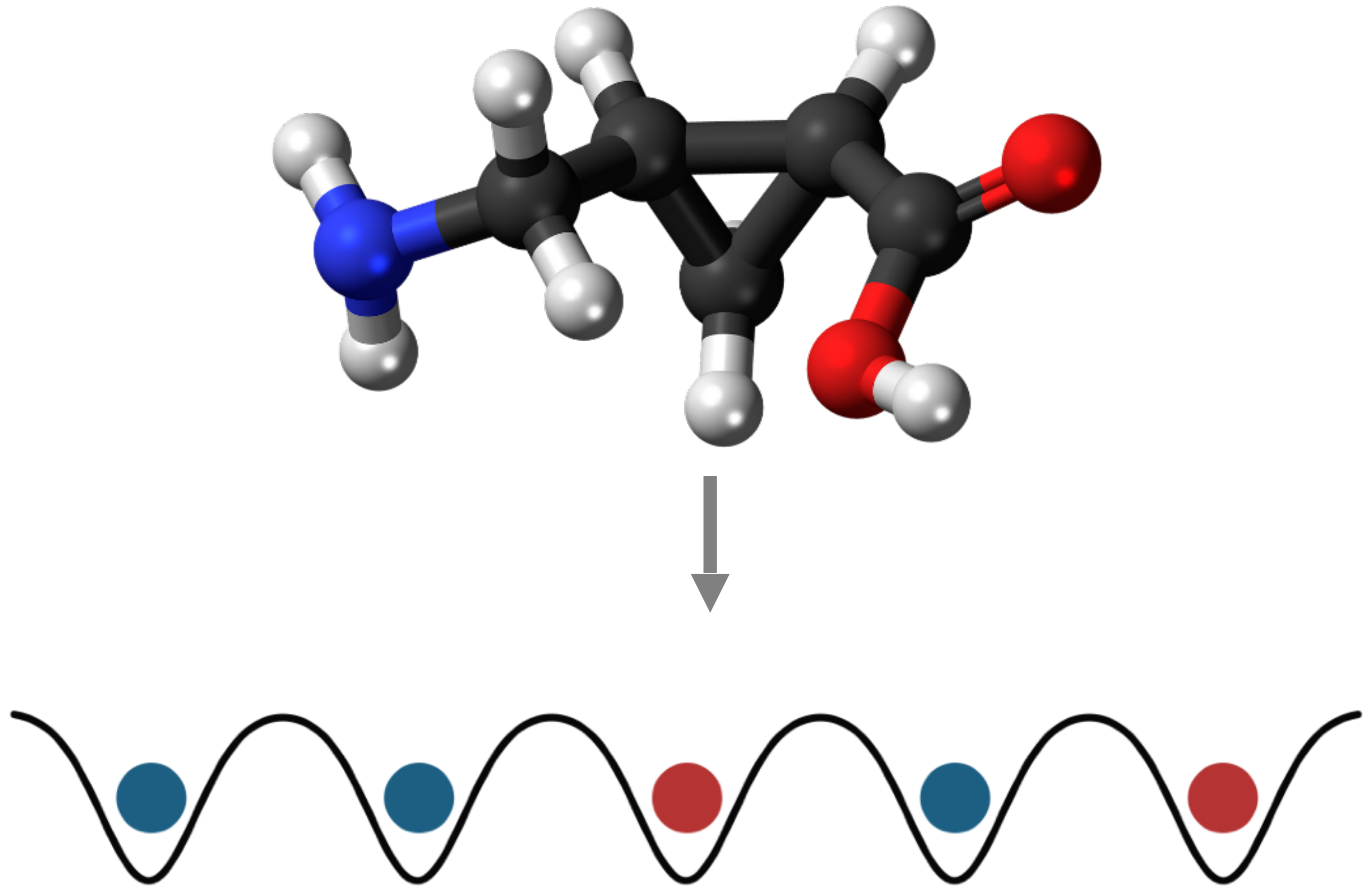
$$\hat{H}_a = \hat{H} + \hbar \sum_{i \le j} \frac{\chi_{ij}}{2} \sqrt{\omega_i \omega_j} (a_i^\dagger a_i + a_j^\dagger a_j + 2 a_i^\dagger a_j^\dagger a_i a_j)$$

- Possible projects for this week

Looks an awful lot like **bosonic** Hubbard problem

C. Sparrow, Nature 557, 660 (2018).

Electron structure



Looks an awful lot like **fermionic** Hubbard problem

J. Argüello-Luengo et al.
Nature 574, 215 (2019).

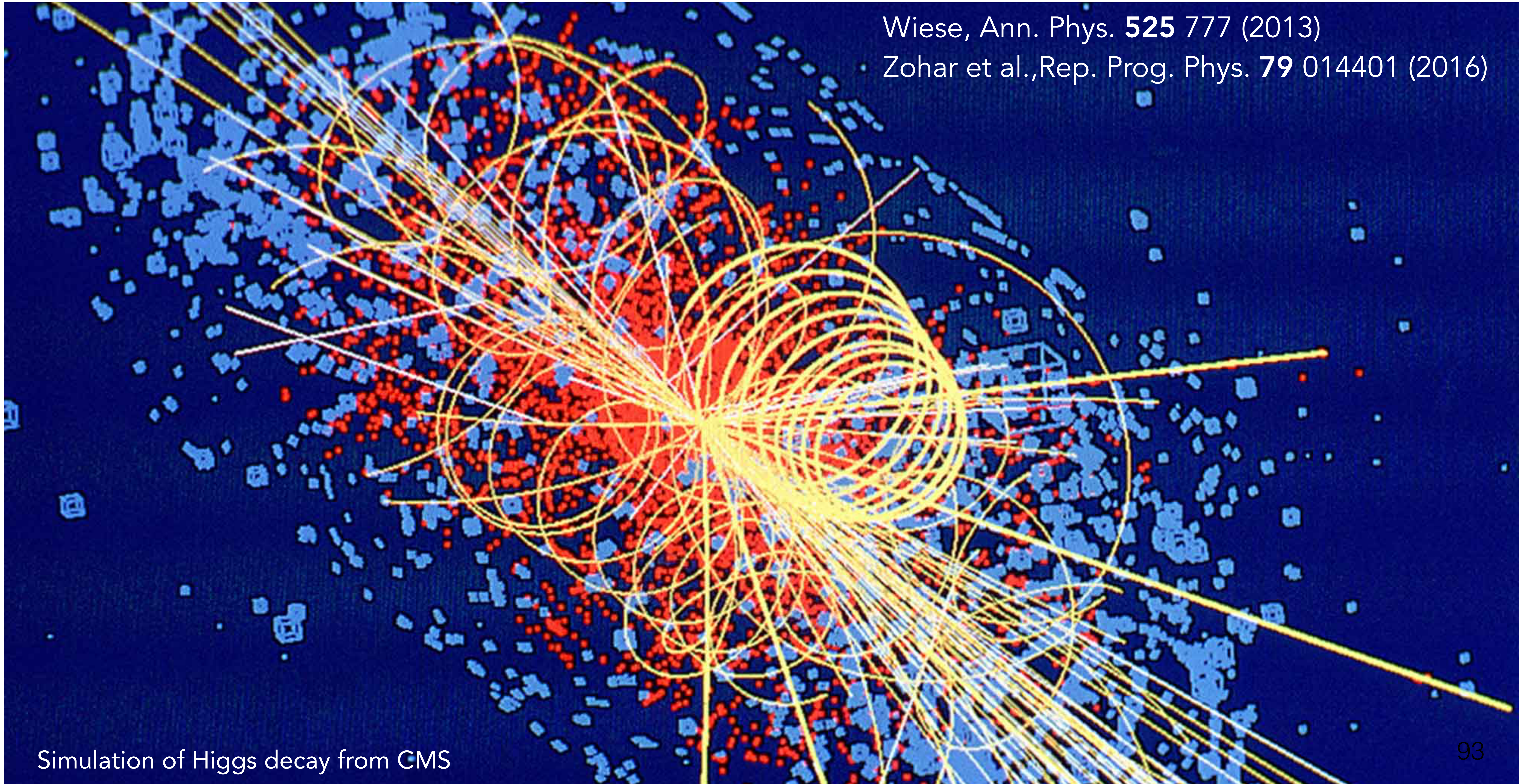
1. Atomic clocks — Qubits in cold atoms
2. Optical tweezers — Trapped qubits in atoms
3. Rydberg atoms — Large scale entanglement
4. Moving particles — Bosons vs Fermions and the link to chemistry
5. Lattice gauge theories — Working on a really hard physics problem

$$\mathcal{L}_{QED} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$\mathcal{L}_{QCD} = \sum_{fi} \bar{\psi}^{fi}\left(i\gamma^{\mu}D_{\mu ij} - m_f\right)\psi^{fi} - \frac{1}{2g^2}Tr\left(G^{\mu\nu}G_{\mu\nu}\right)$$

Particle

Gauge field



Wiese, Ann. Phys. **525** 777 (2013)

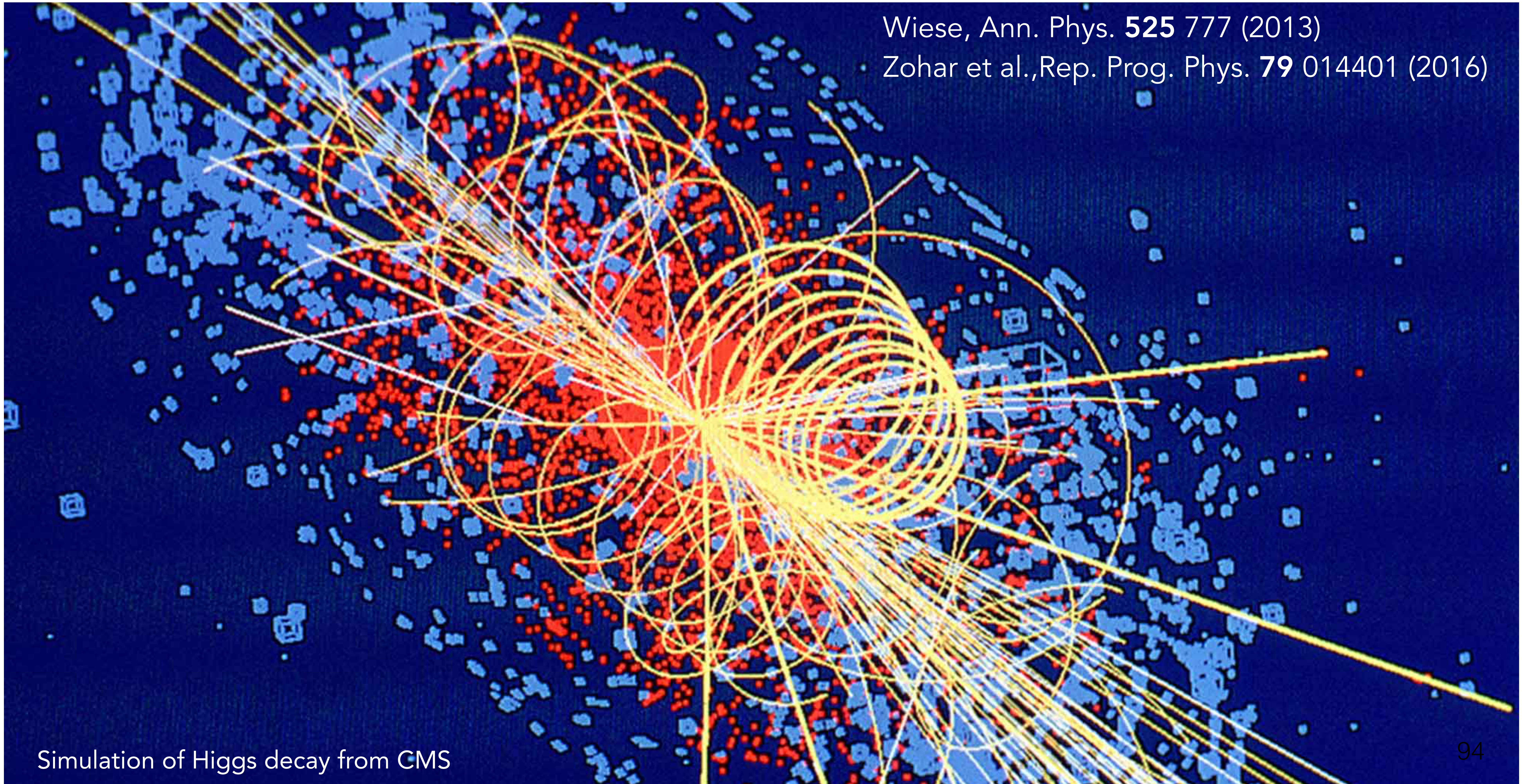
Zohar et al.,Rep. Prog. Phys. **79** 014401 (2016)

Simulation of Higgs decay from CMS

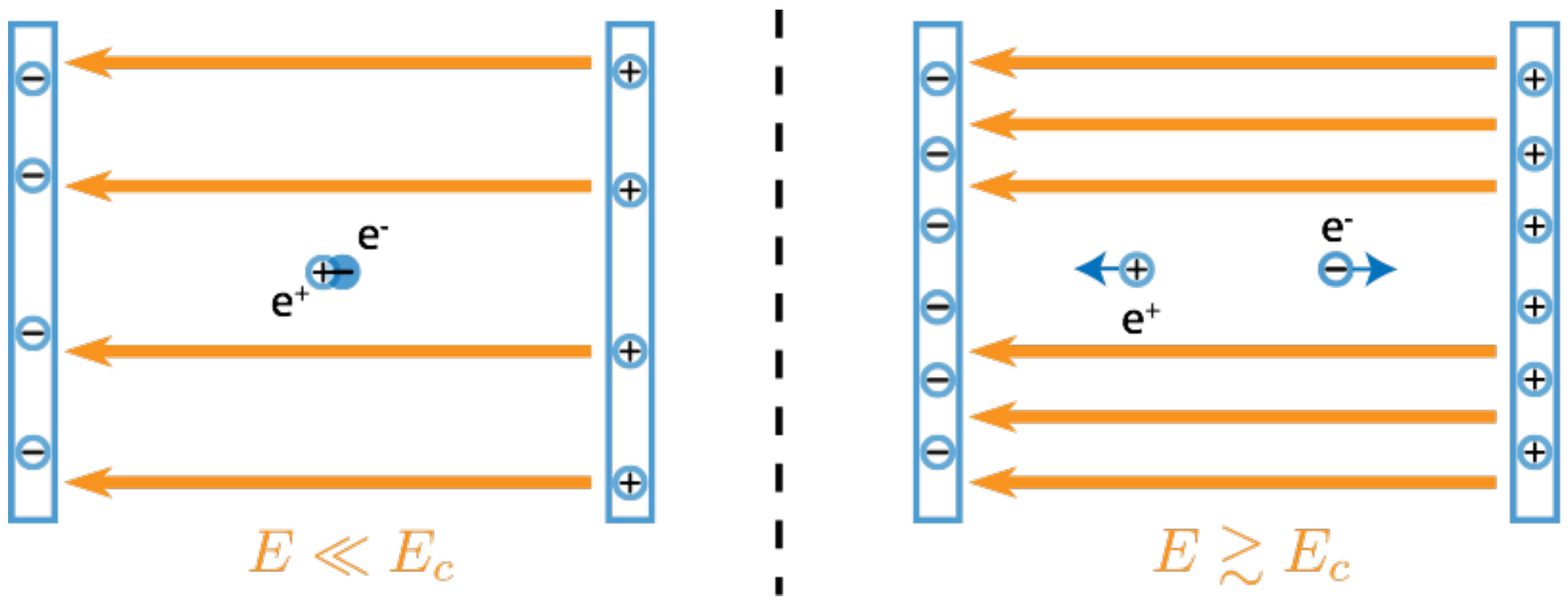
$$\mathscr{L}_{QED} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$\mathscr{L}_{QCD} = \sum_{fi} \bar{\psi}^{fi}\left(i\gamma^{\mu}D_{\mu ij} - m_f\right)\psi^{fi} - \frac{1}{2g^2}Tr\left(G^{\mu\nu}G_{\mu\nu}\right)$$

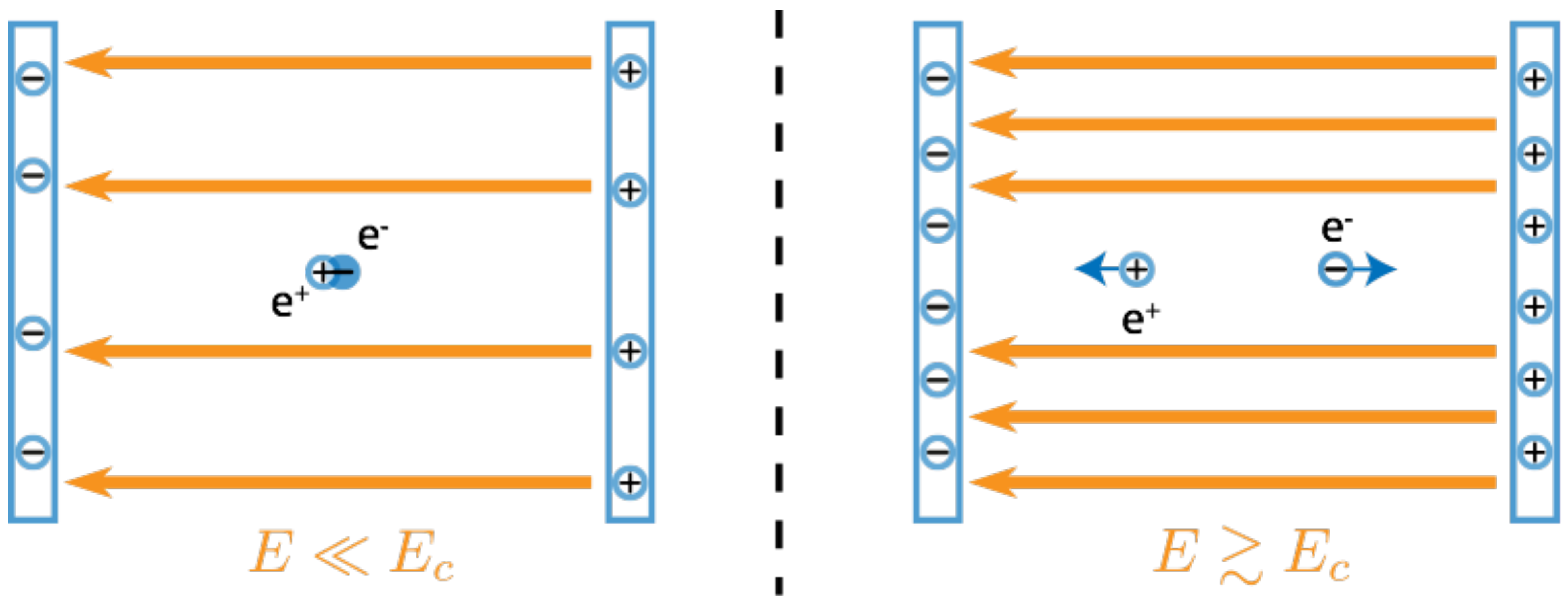
Particle Gauge coupling Gauge field



J. Schwinger, Phys. Rev. **714**, 16 (1951).

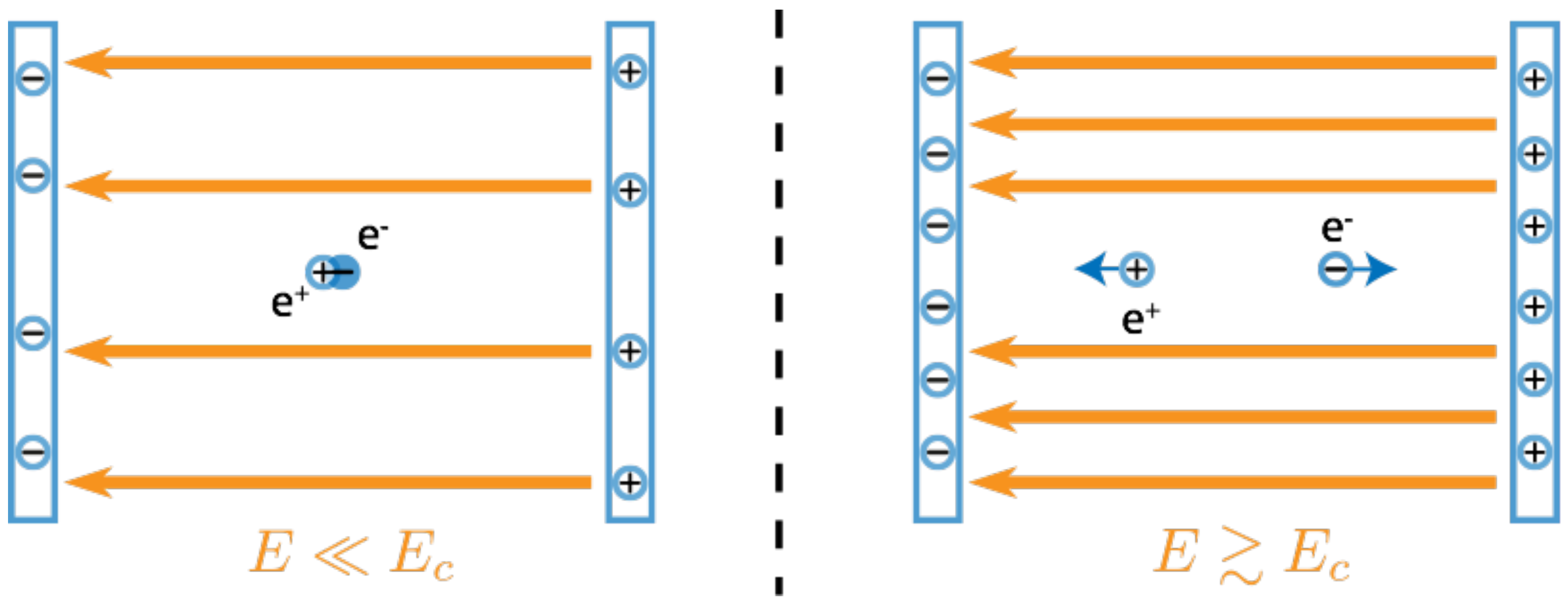


J. Schwinger, Phys. Rev. **714**, 16 (1951).



$$E_c = \frac{m_e^2 c^3}{\hbar q_e} \approx 10^{18} \mathrm{V/m}$$

J. Schwinger, Phys. Rev. **714**, 16 (1951).

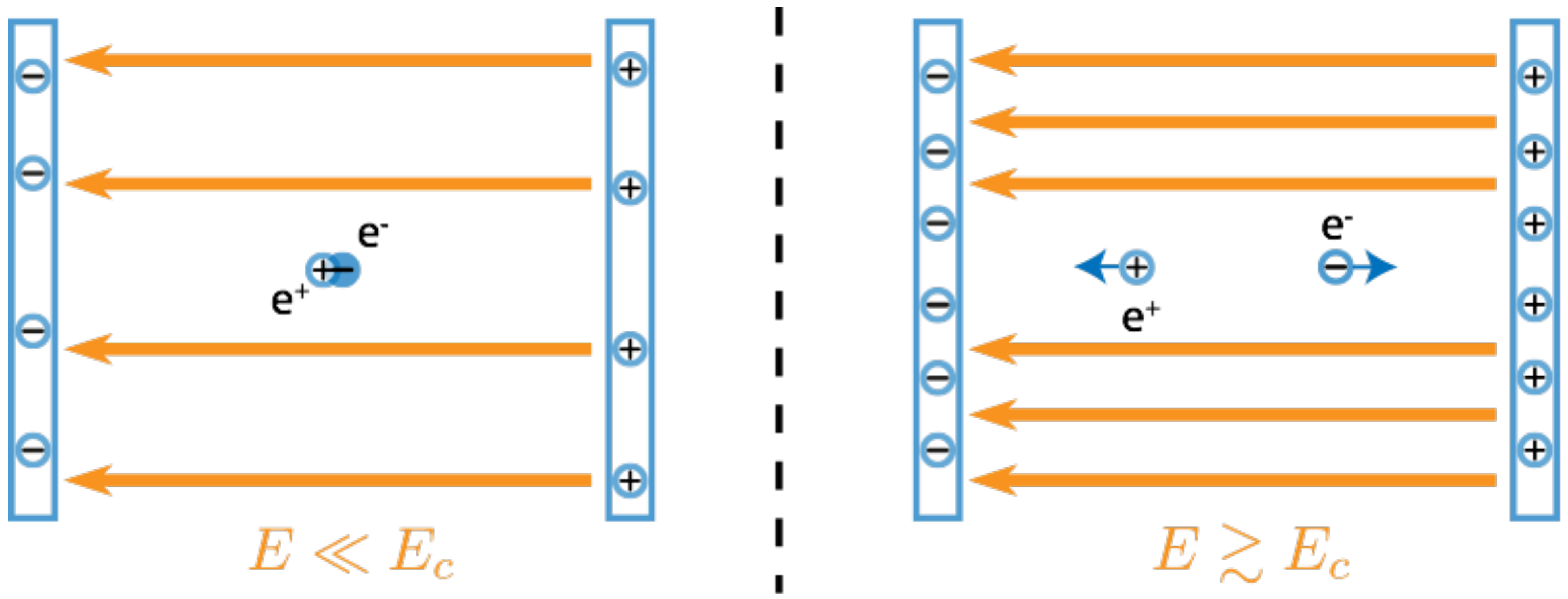


$$E_c = \frac{m_e^2 c^3}{\hbar q_e} \approx 10^{18} \mathrm{V/m}$$

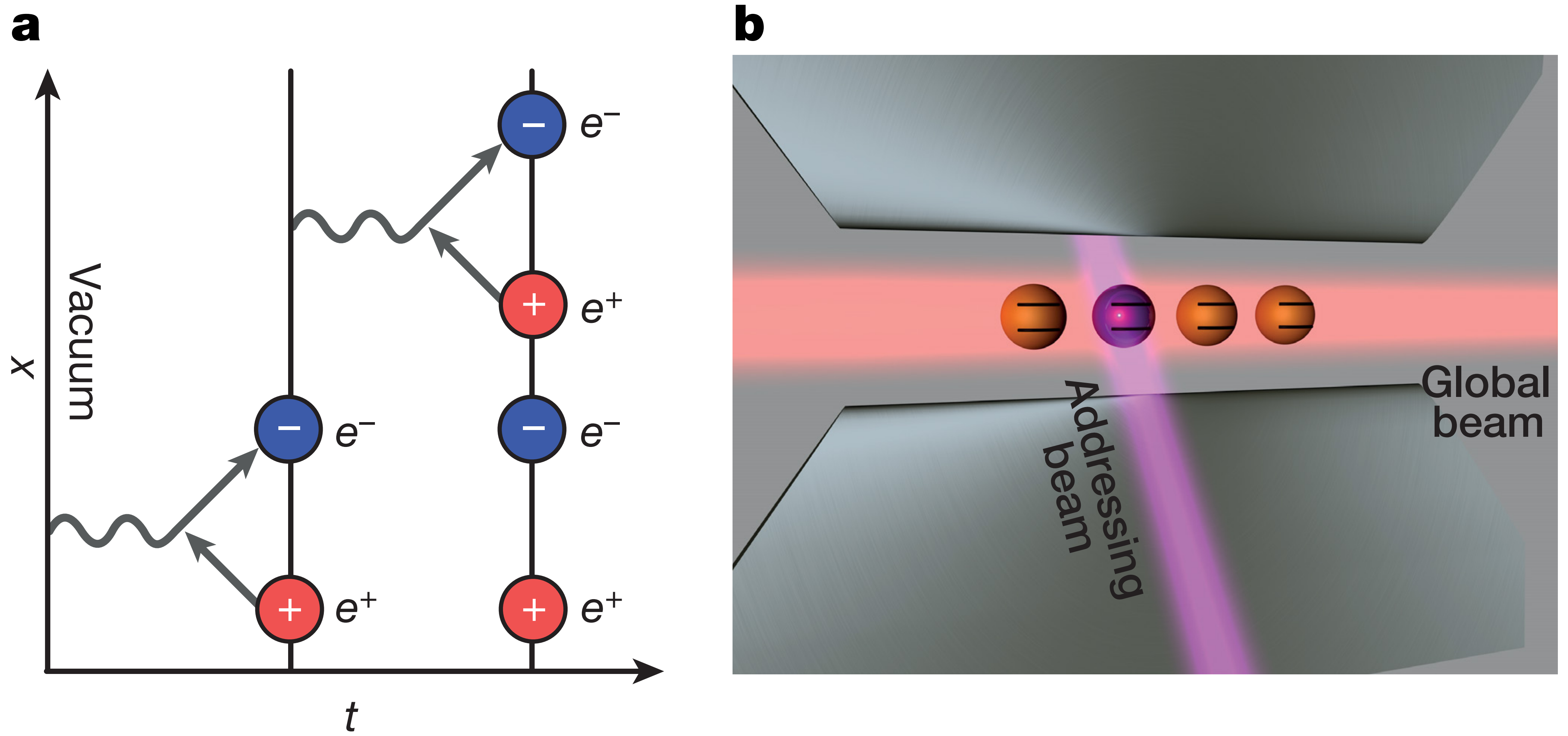
Can we construct a quantum simulator ?

10^5 10^9 10^{18}

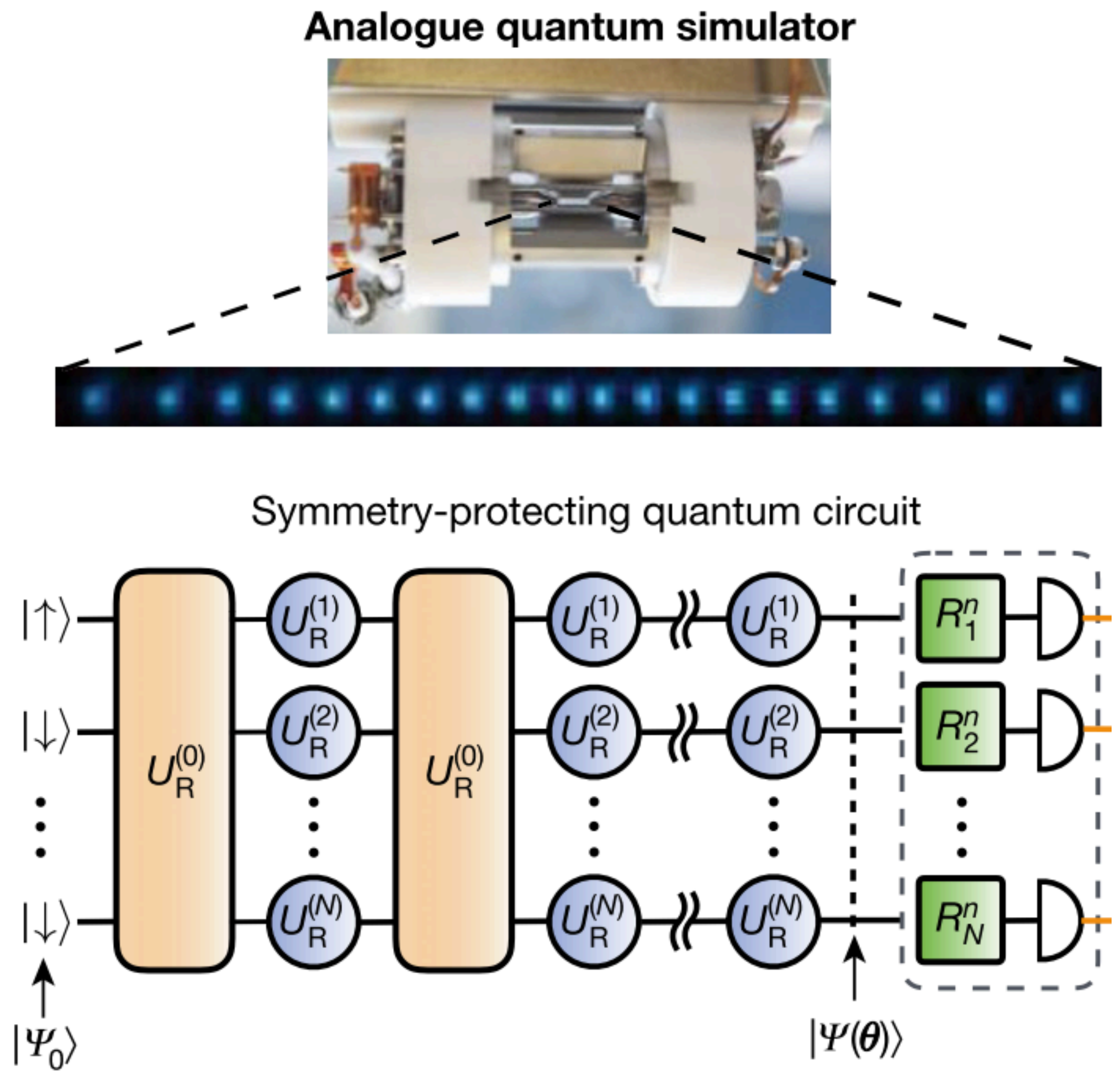
Log E



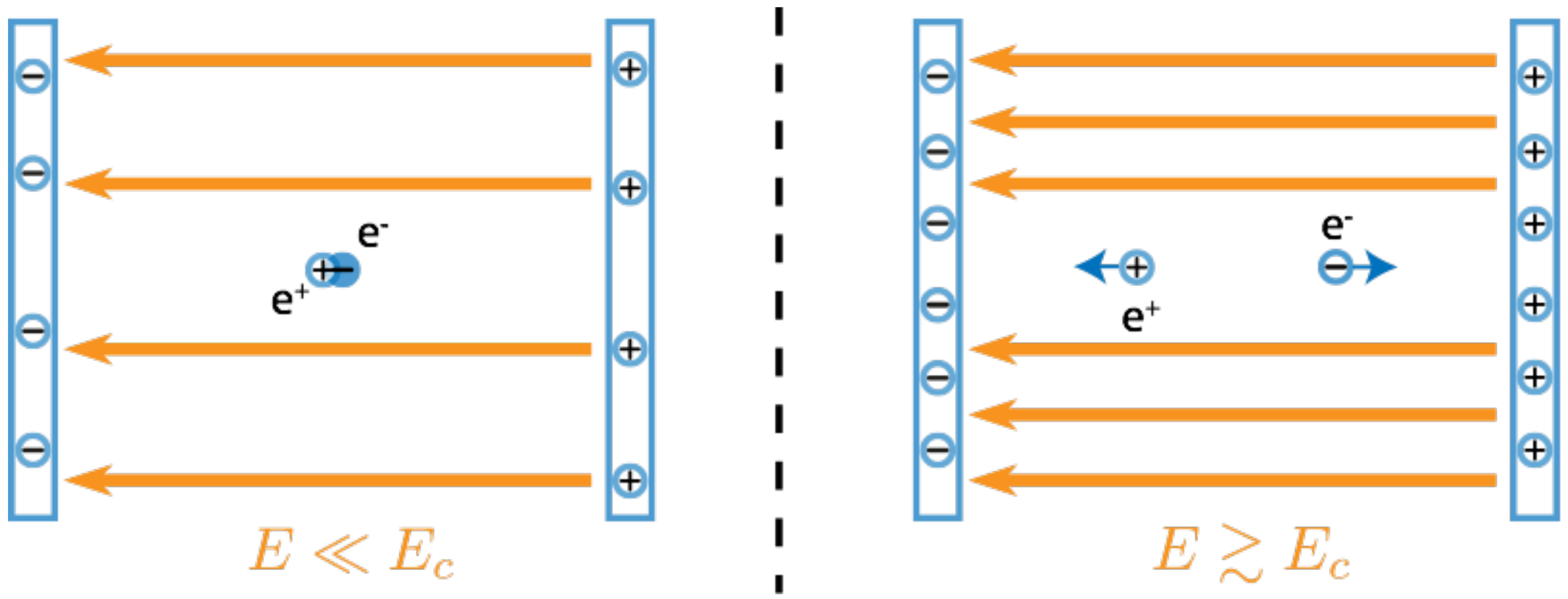
First digital
implementation with ions



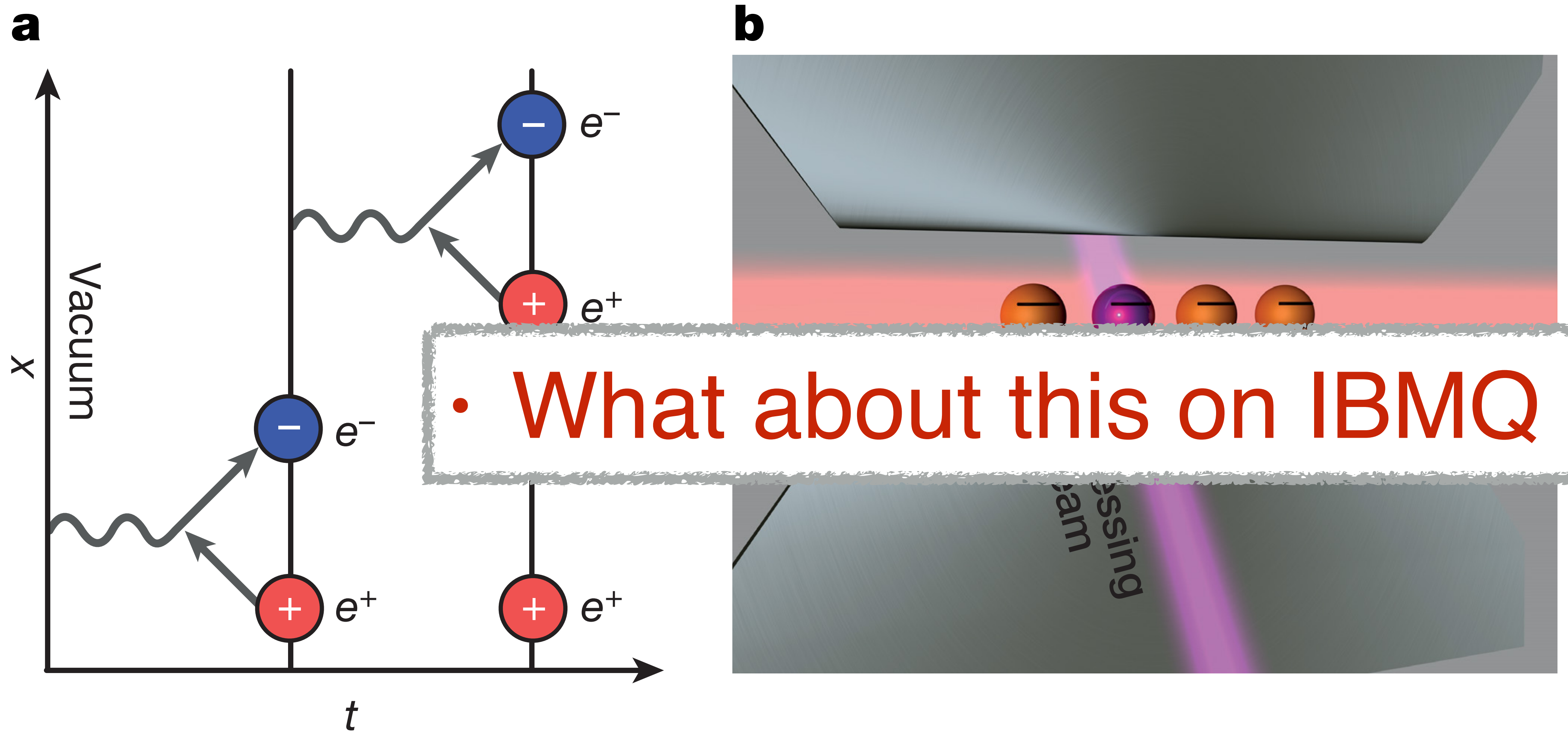
E. Martinez et al., Nature **534** 516 (2016).



C. Kokail et al., Nature **569**, 355 (2019).

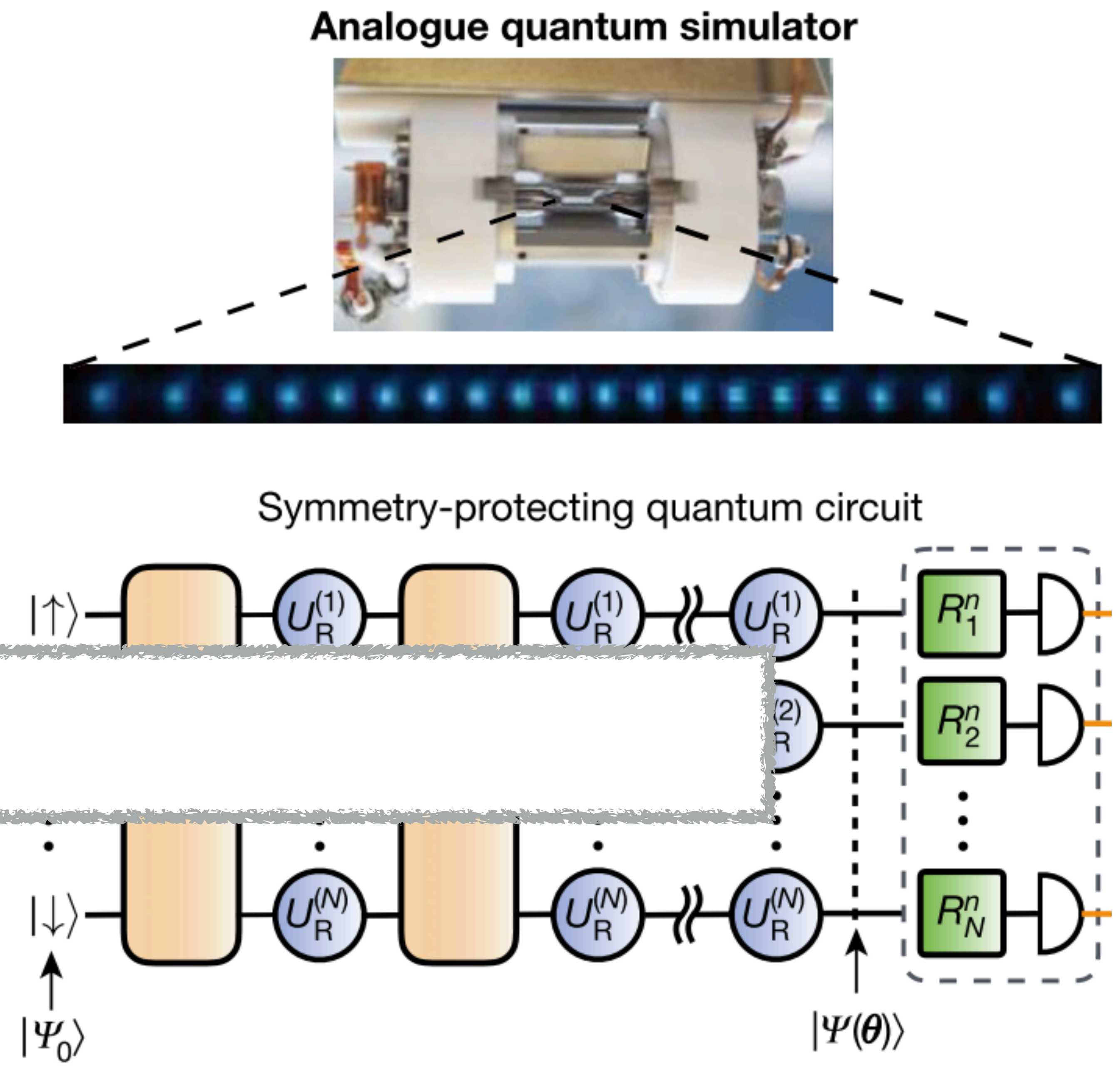


First digital implementation with ions

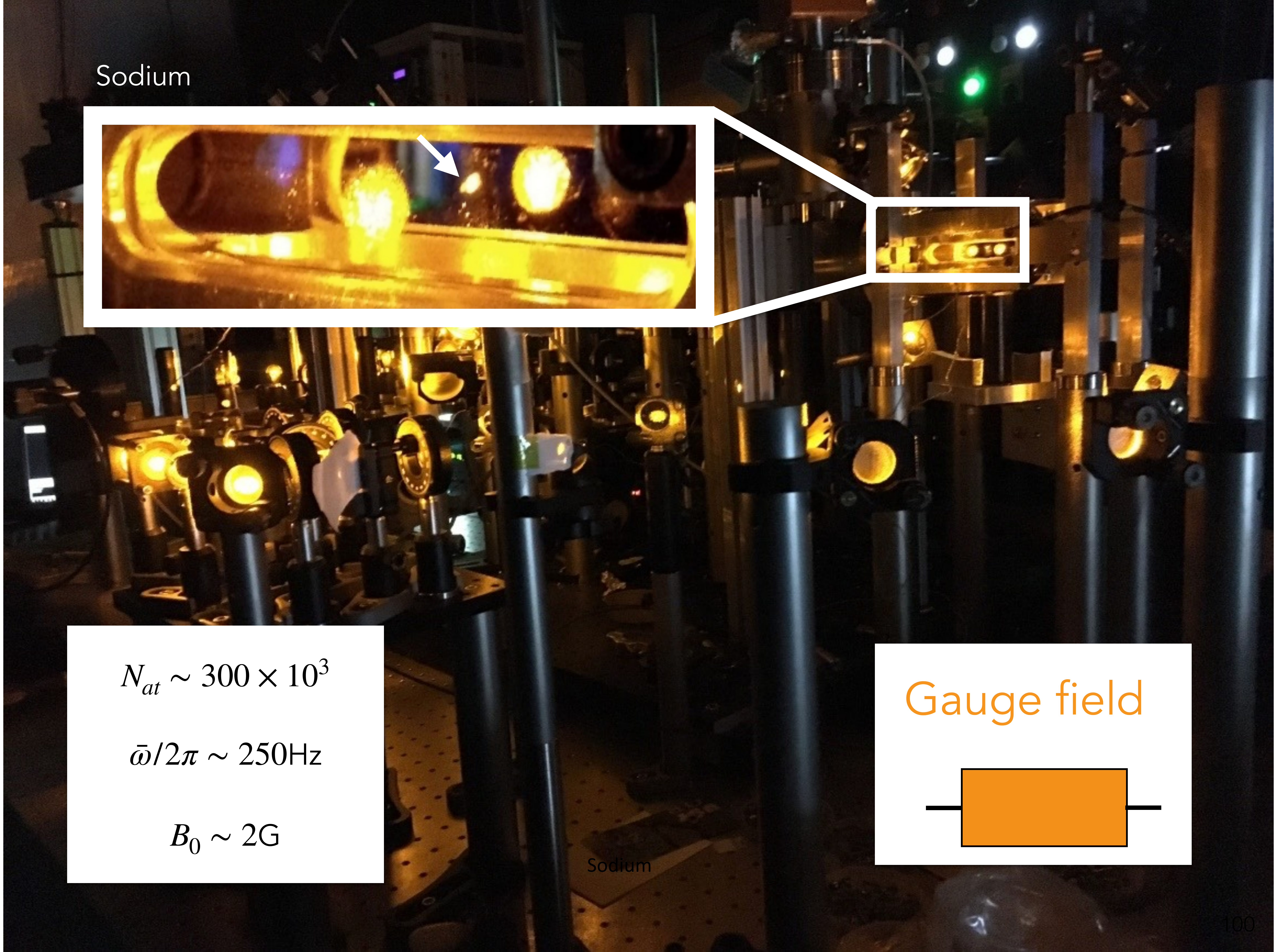


- What about this on IBMQ ?

E. Martinez et al., Nature **534** 516 (2016).



C. Kokail et al., Nature **569**, 355 (2019).



Sodium

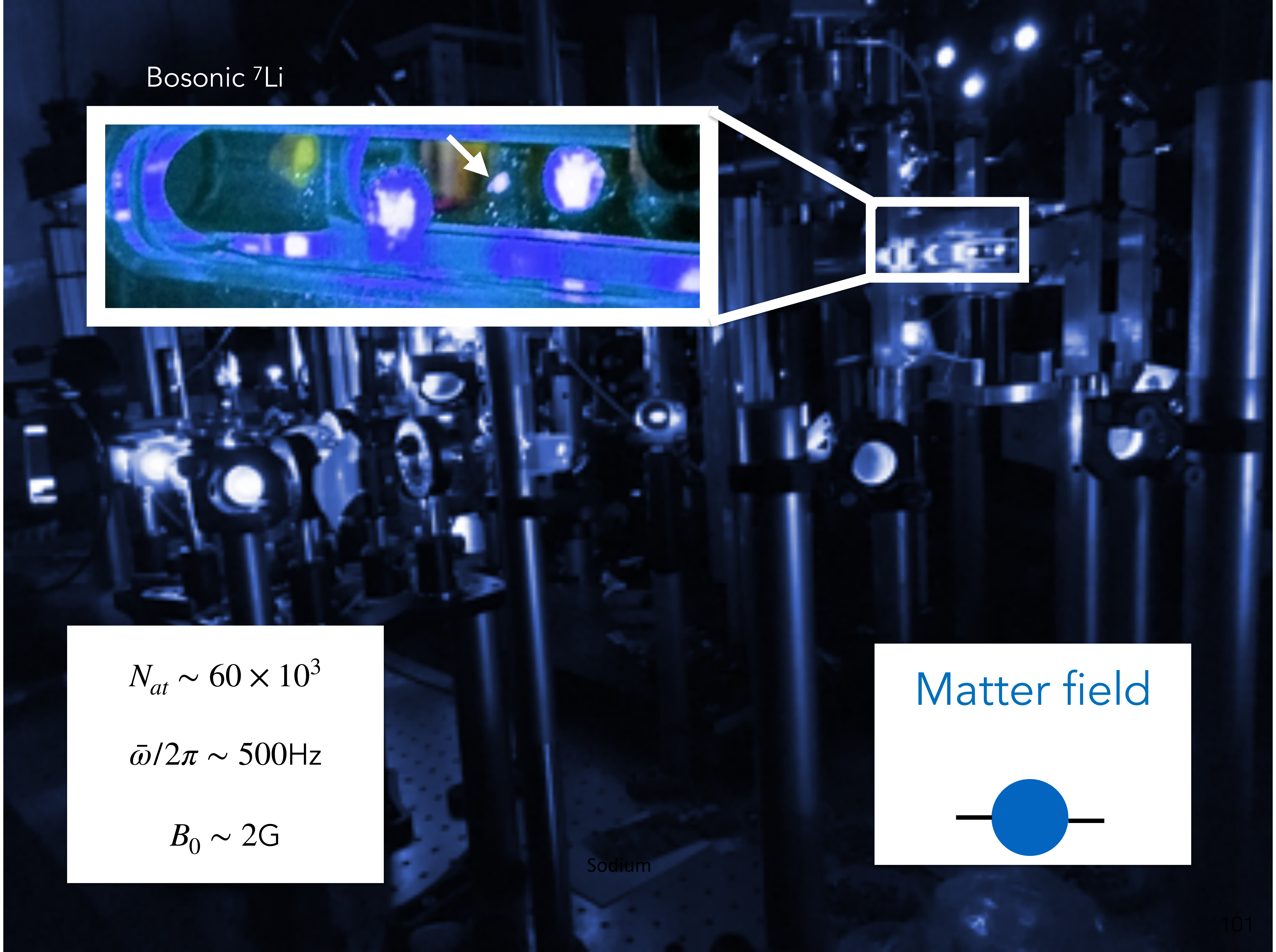
$N_{at} \sim 300 \times 10^3$

$\bar{\omega}/2\pi \sim 250\text{Hz}$

$B_0 \sim 2\text{G}$

Gauge field

Sodium

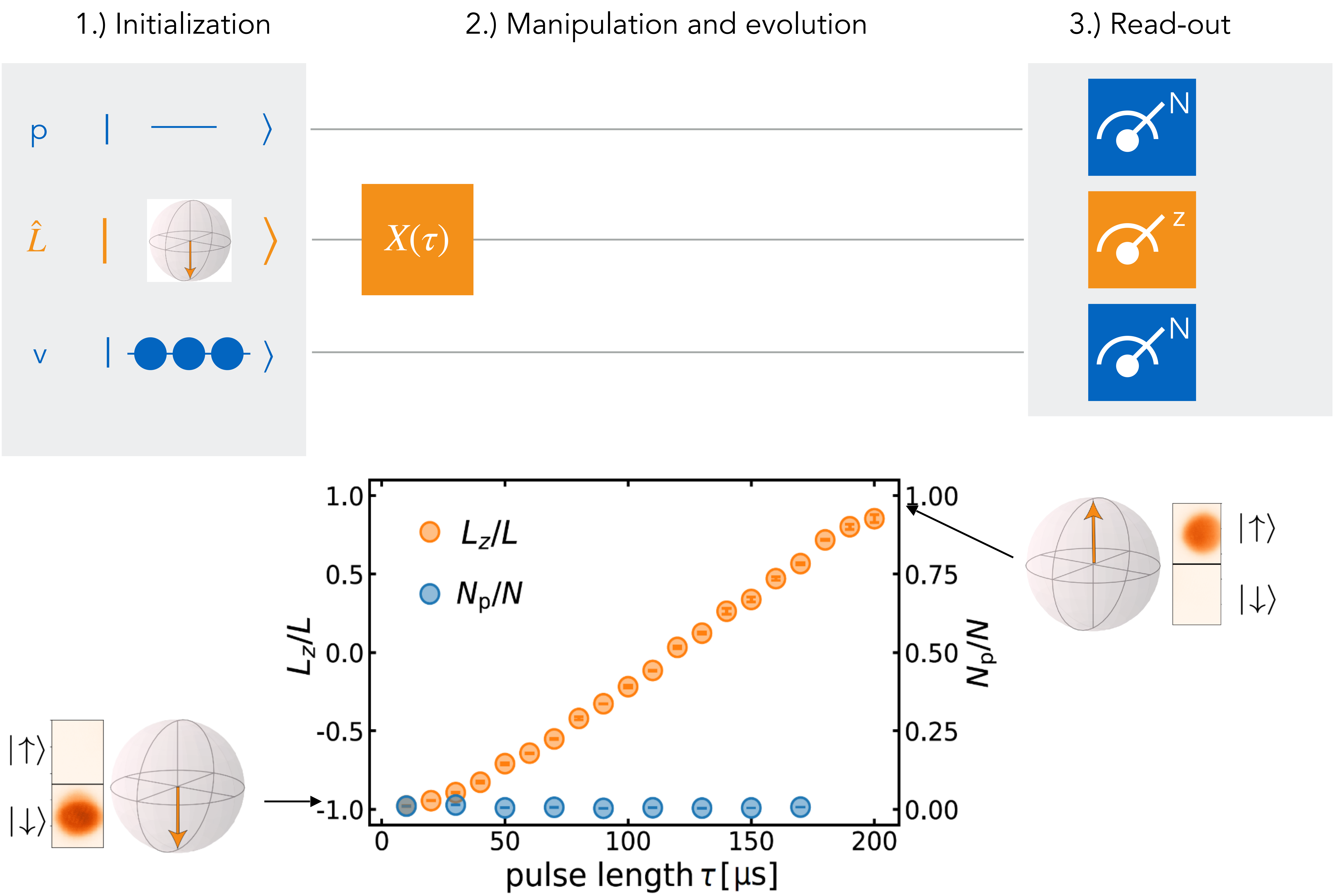

Bosonic ^{7}Li
$N_{at} \sim 60 \times 10^3$
$\bar{\omega}/2\pi \sim 500\text{Hz}$
$B_0 \sim 2\text{G}$
Sodium
Matter field

1.) Initialization

2.) Manipulation and evolution

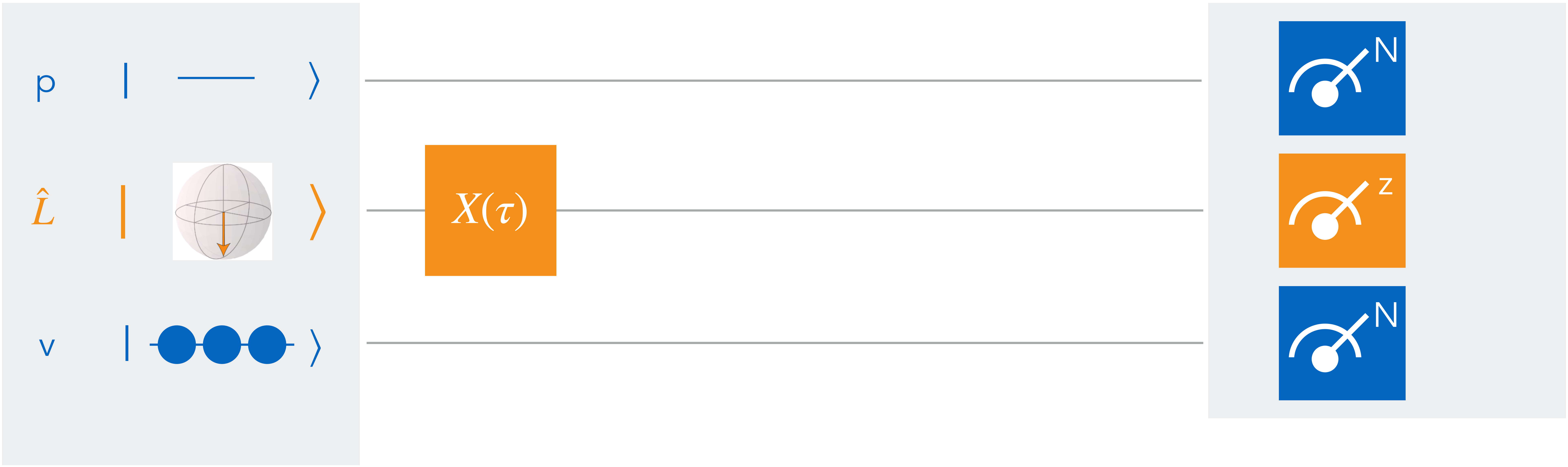
3.) Read-out

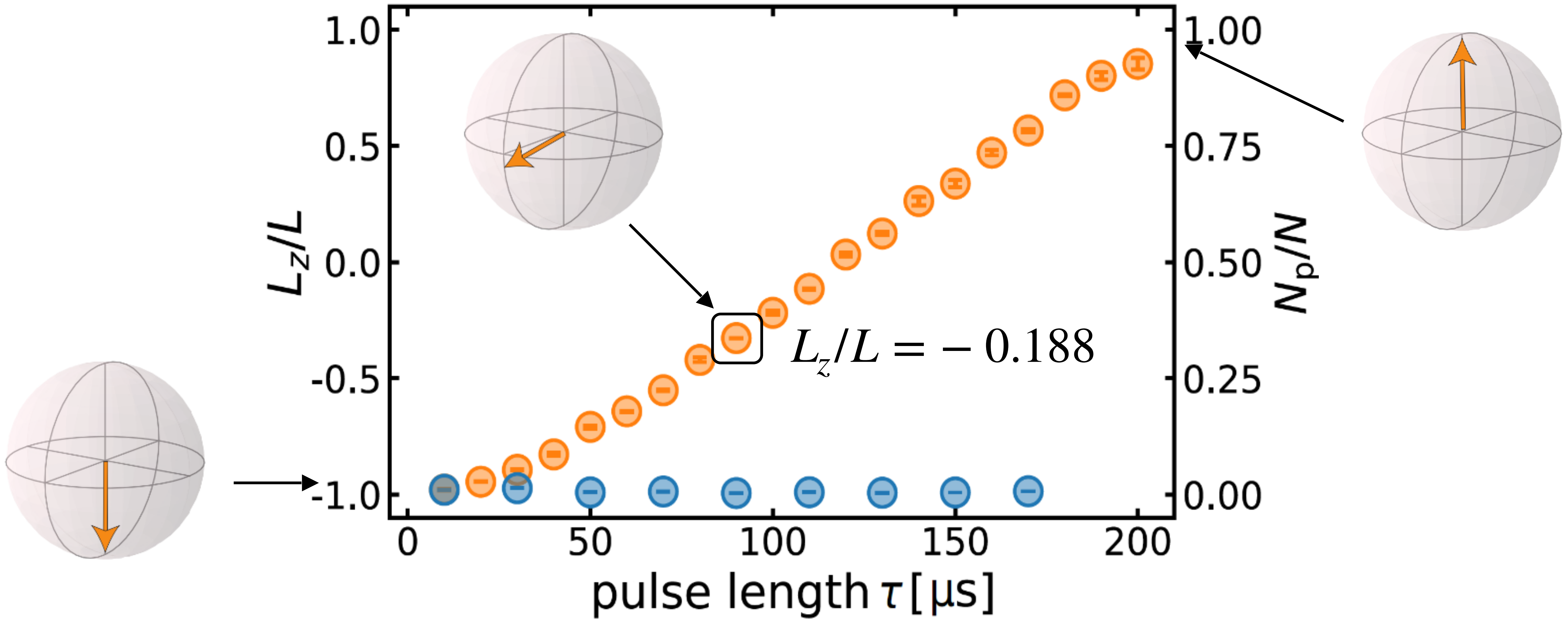
Mil et al. Science **367**, 1128 (2020)



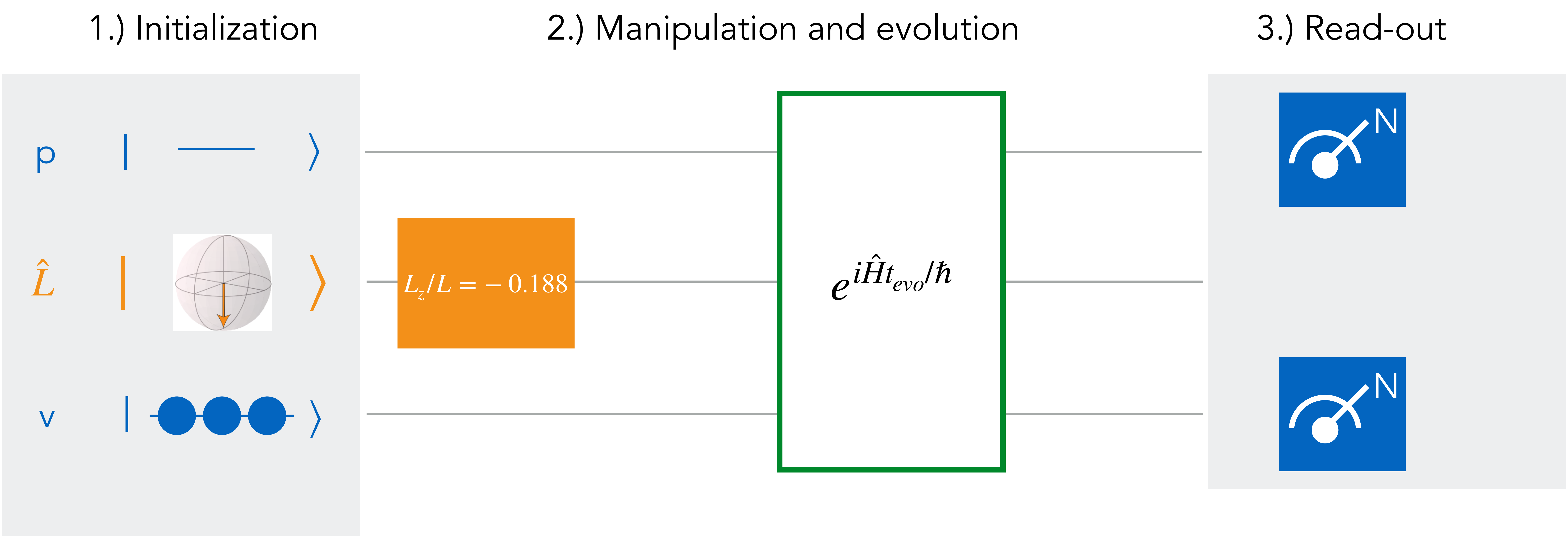
Mil et al. Science **367**, 1128 (2020)

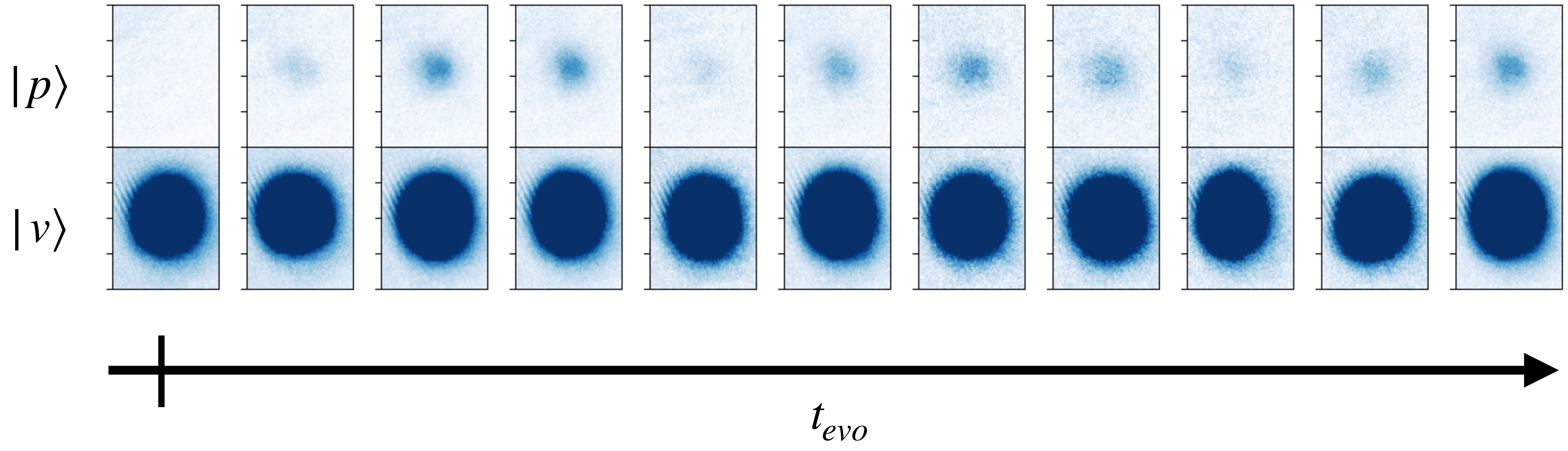
1.) Initialization 2.) Manipulation and evolution 3.) Read-out



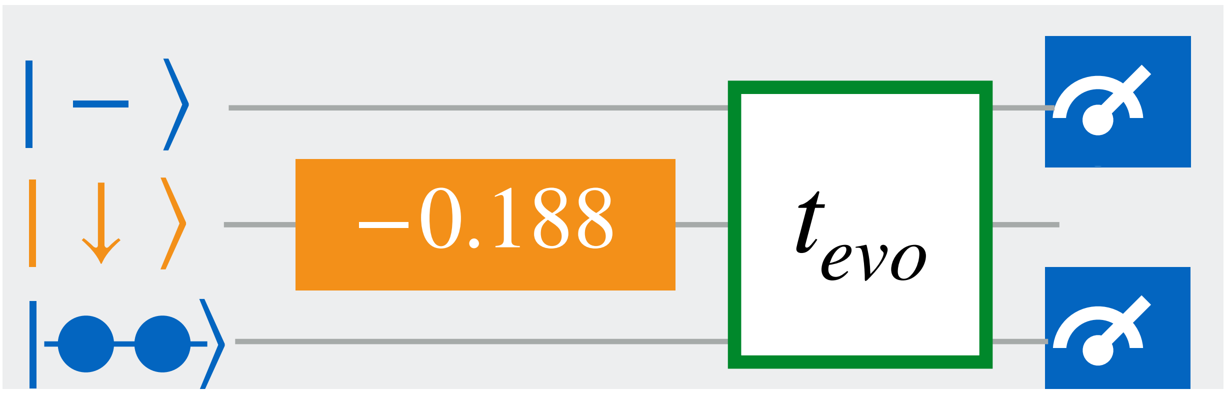


Mil et al. Science **367**, 1128 (2020)

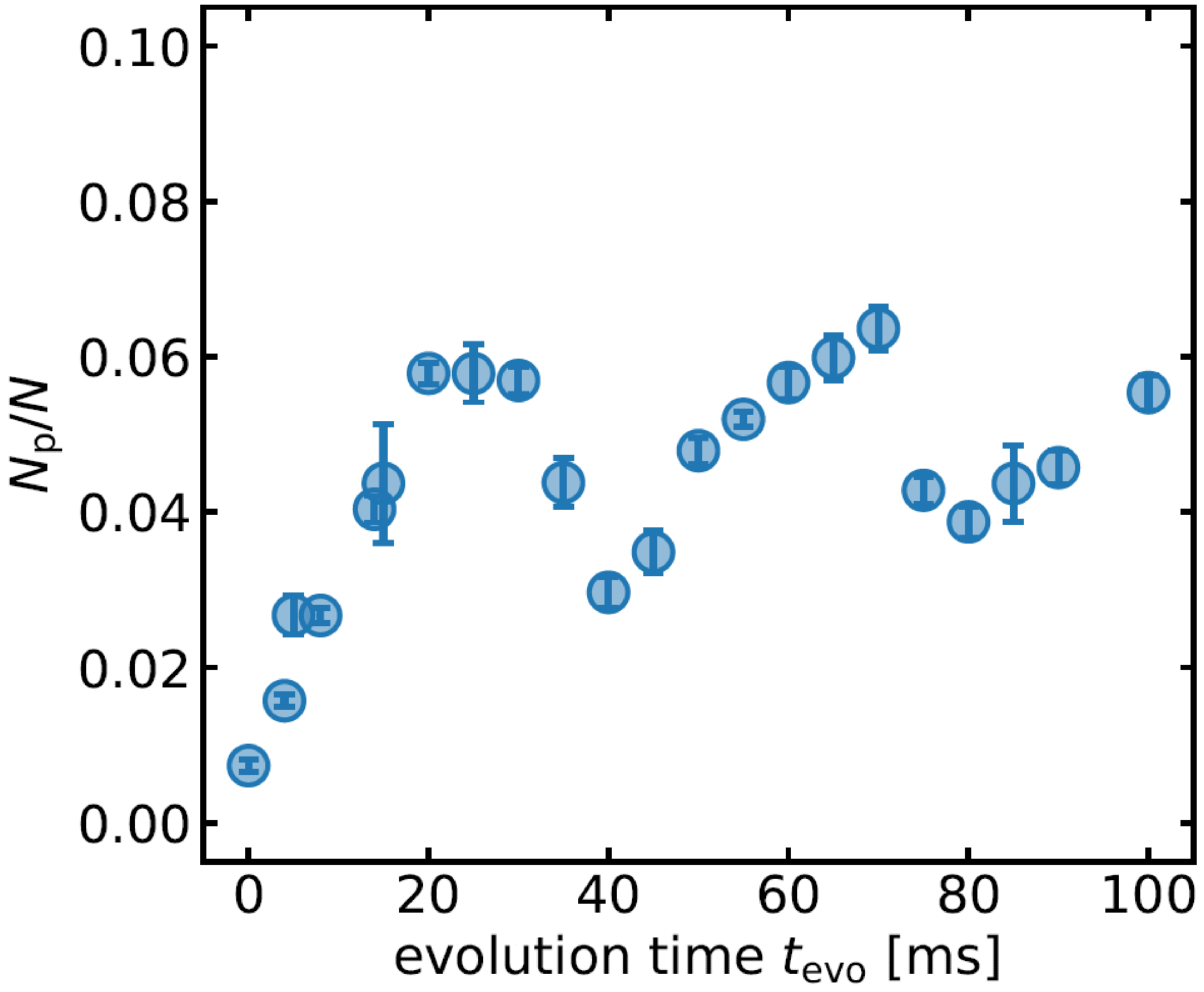




Mil et al. Science **367**, 1128 (2020)

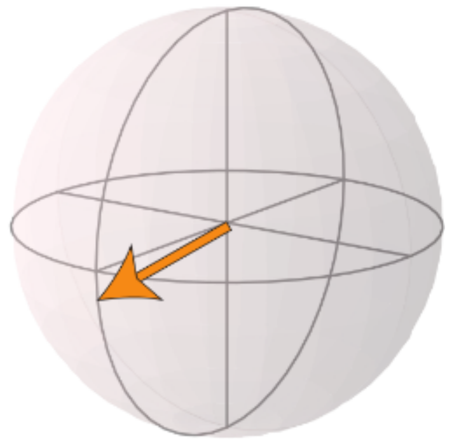


|−⟩
|↓⟩
−0.188
t_{evo}

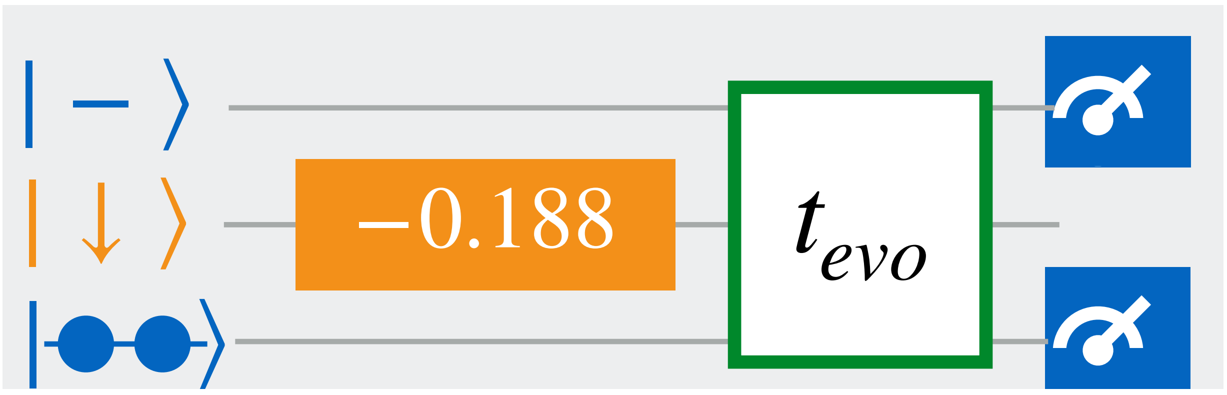


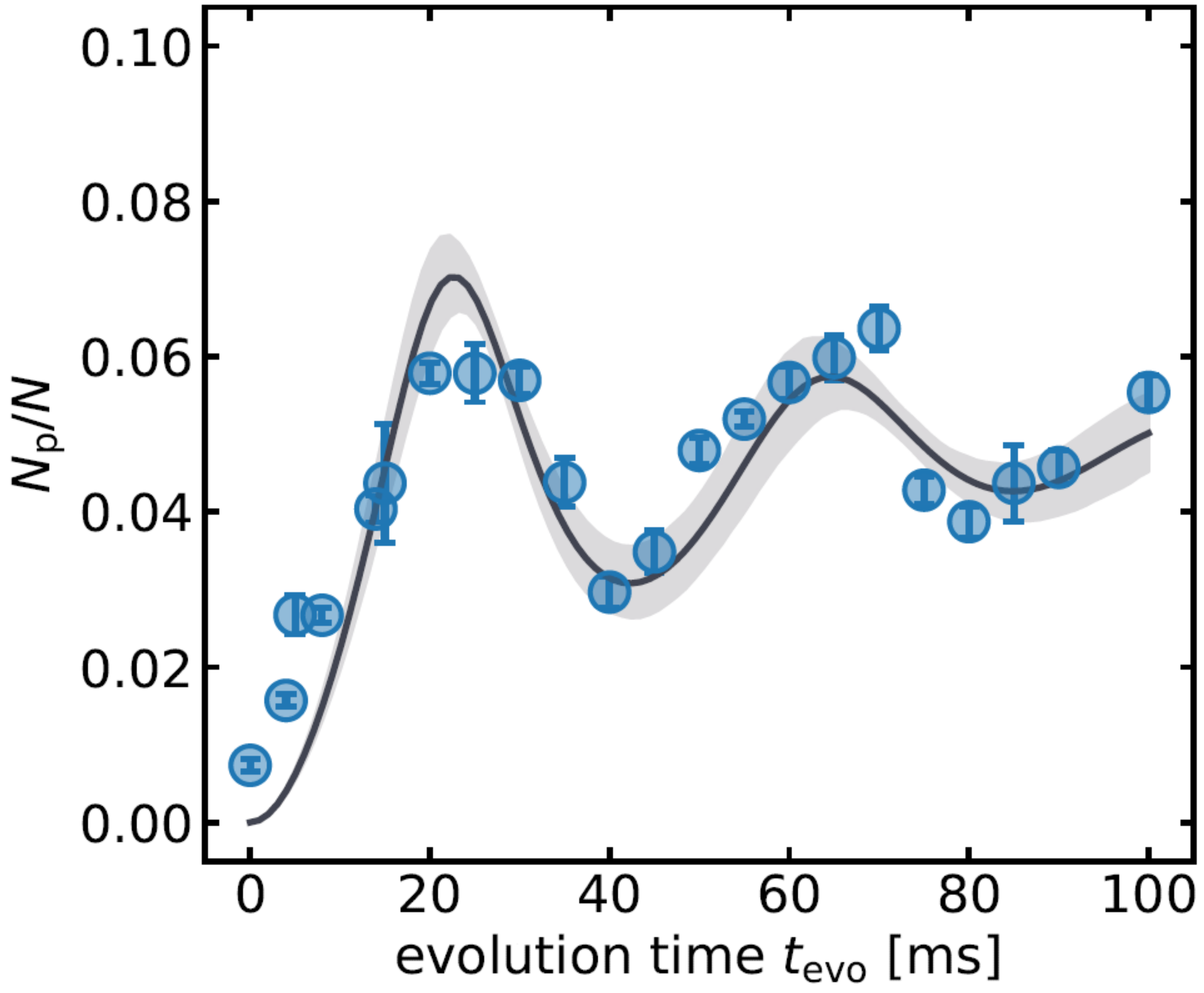
0.10
0.08
0.06
0.04
0.02
0.00
N_p/N
0
20
40
60
80
100
evolution time t_{evo} [ms]

L_z/L = -0.188

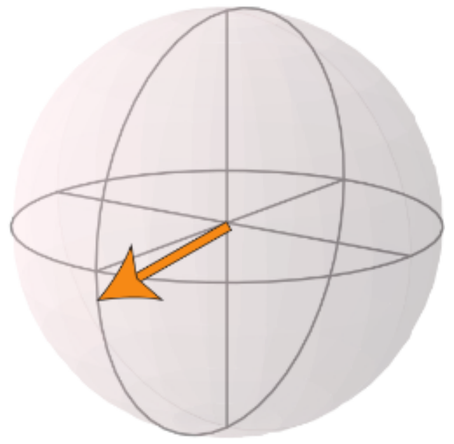


$$\hat{H}/\hbar = \chi \hat{L}_z^2 + \frac{\Delta}{2}\left(\hat{b}_p^\dagger \hat{b}_p - \hat{b}_v^\dagger \hat{b}_v\right) + \lambda\left(b_v^\dagger \hat{L}_- \hat{b}_v + b_v^\dagger \hat{L}_+ \hat{b}_p\right)$$

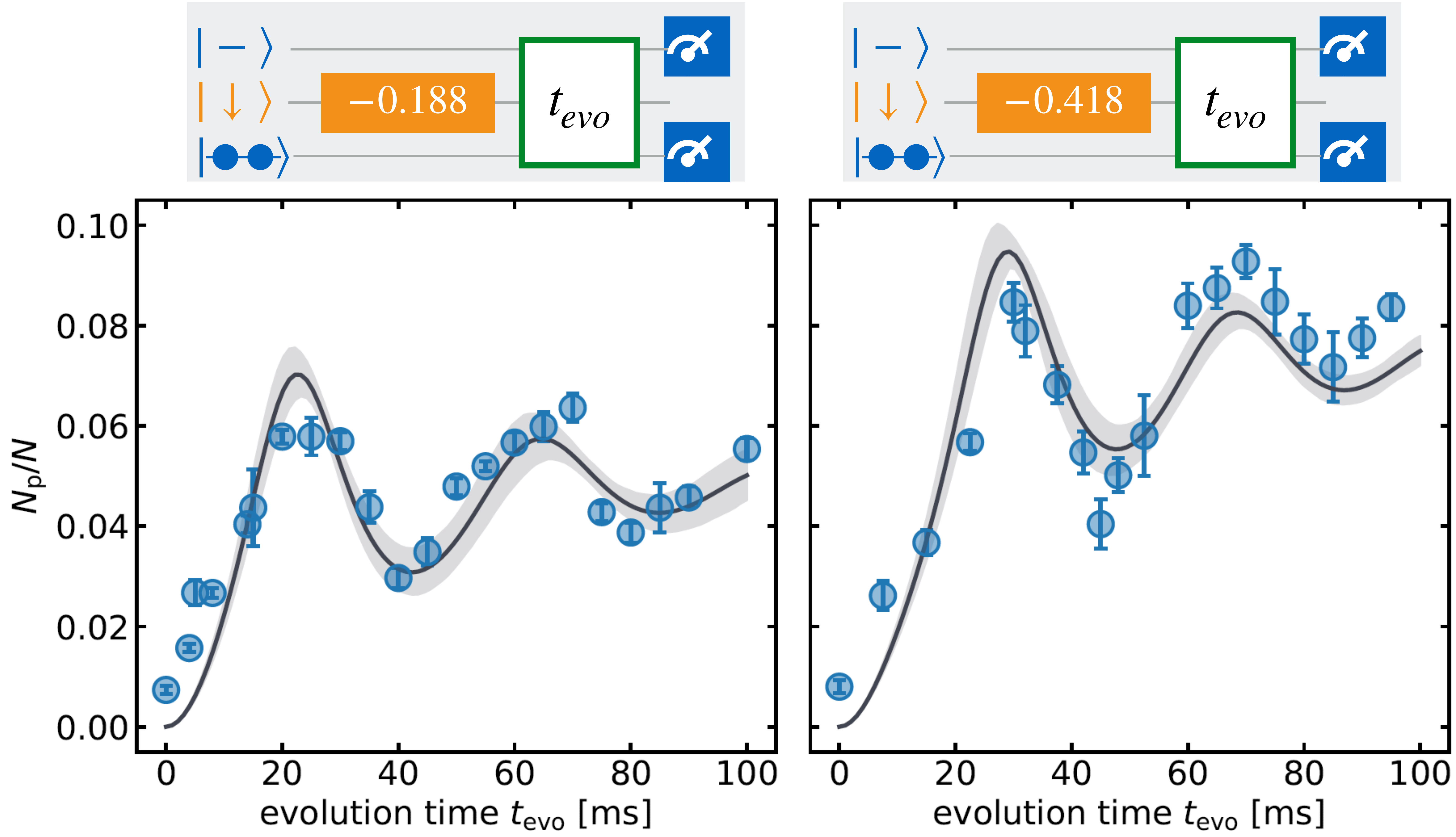




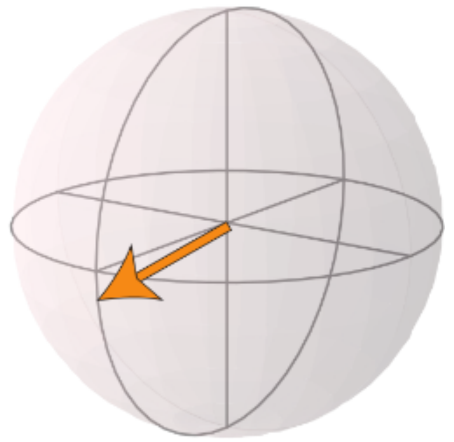
L_z/L = -0.188



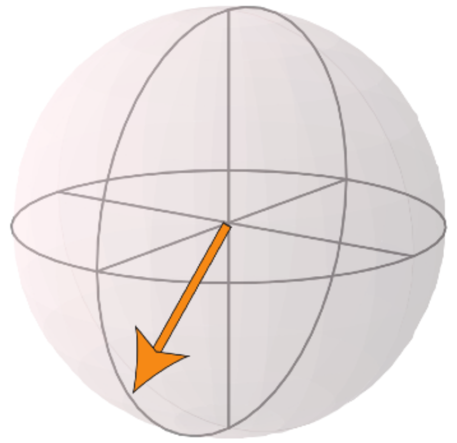
$$\hat{H}/\hbar = \chi \hat{L}_z^2 + \frac{\Delta}{2}\left(\hat{b}_p^\dagger \hat{b}_p - \hat{b}_v^\dagger \hat{b}_v\right) + \lambda\left(b_v^\dagger \hat{L}_- \hat{b}_v + b_v^\dagger \hat{L}_+ \hat{b}_p\right)$$

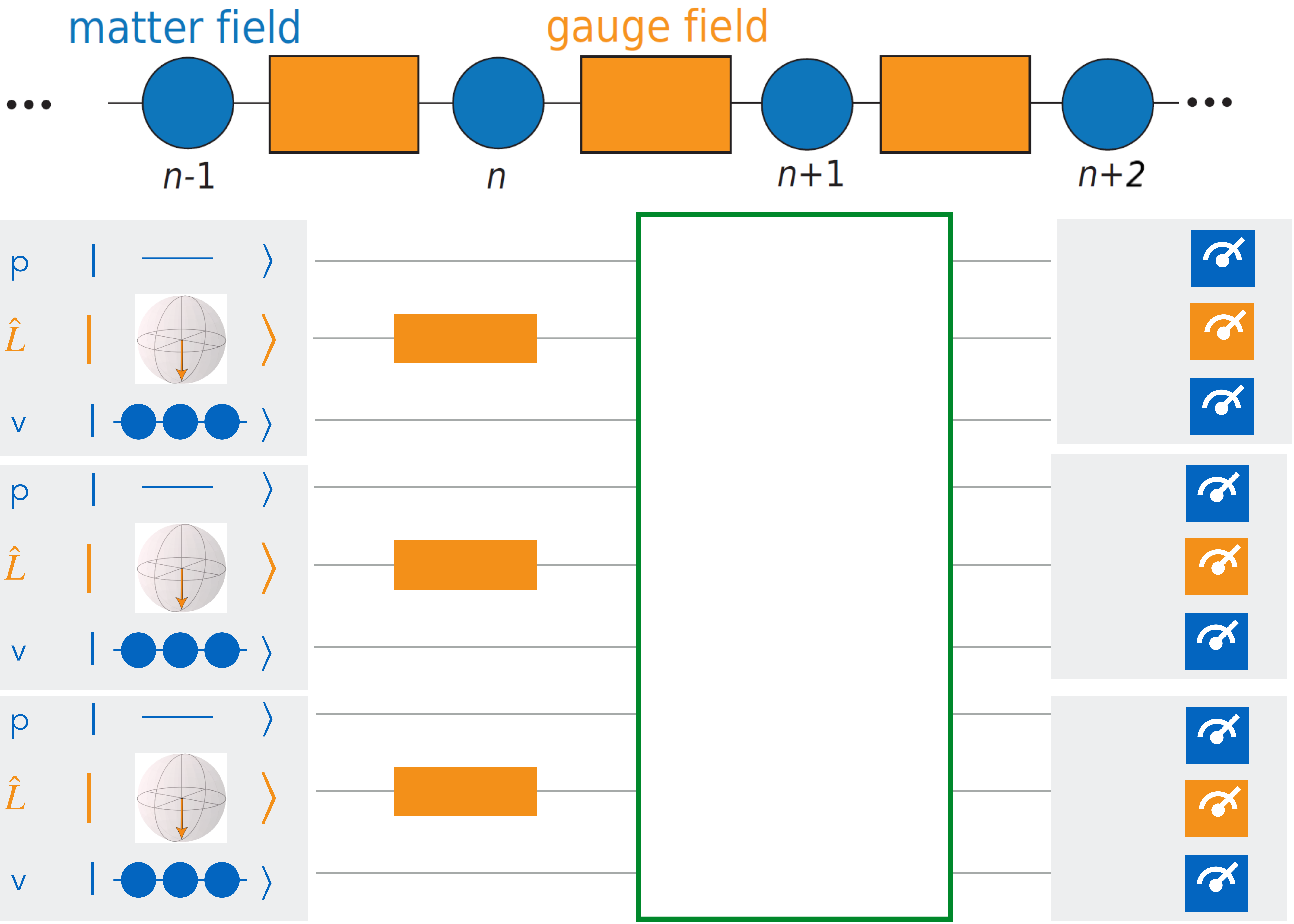


L_z/L = -0.188



L_z/L = -0.418





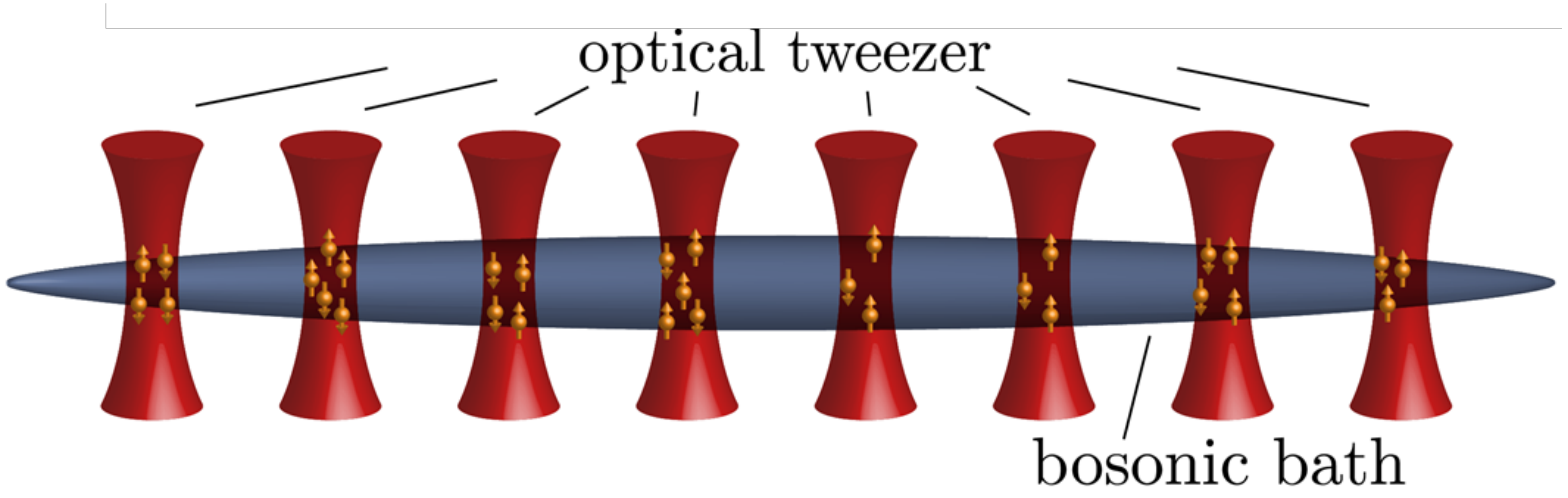
1.) Initialization

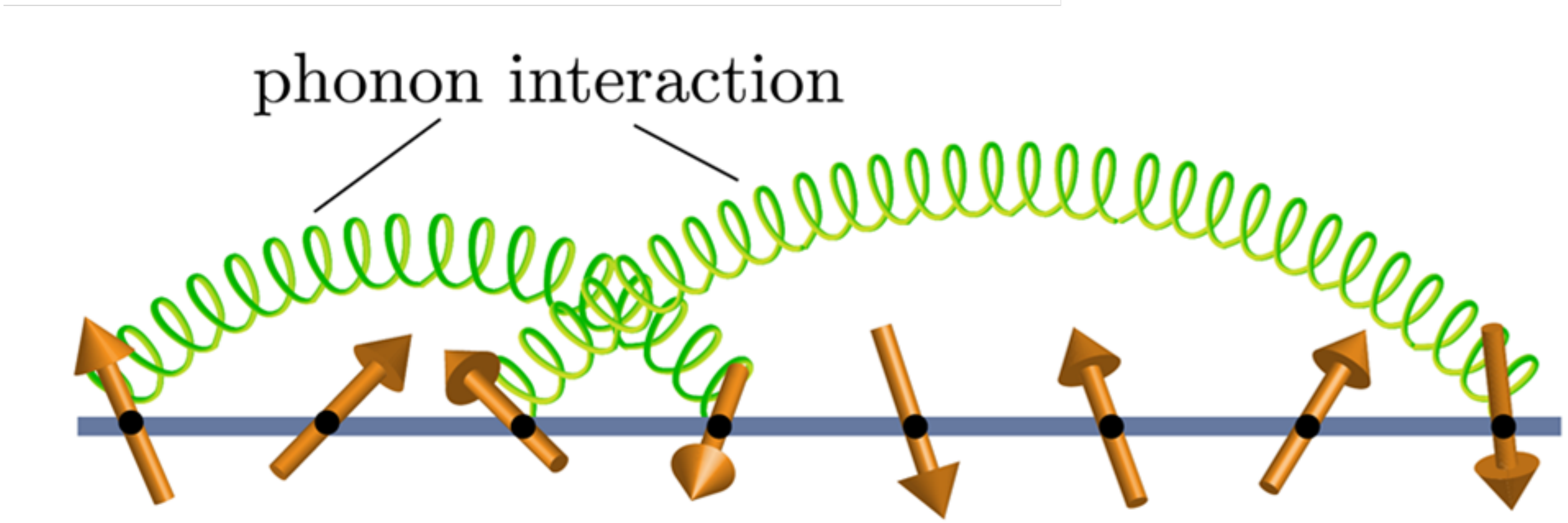
2.) Manipulation and evolution

3.) Read-out

Mil et al. Science **367**, 1128 (2020)

Universal QC with atomic mixtures

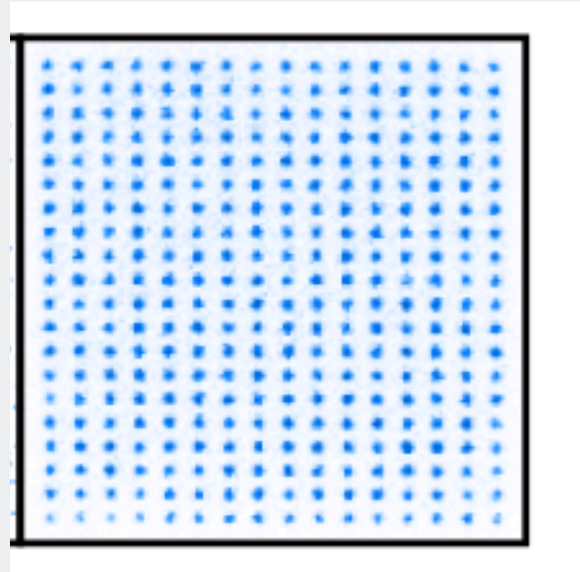
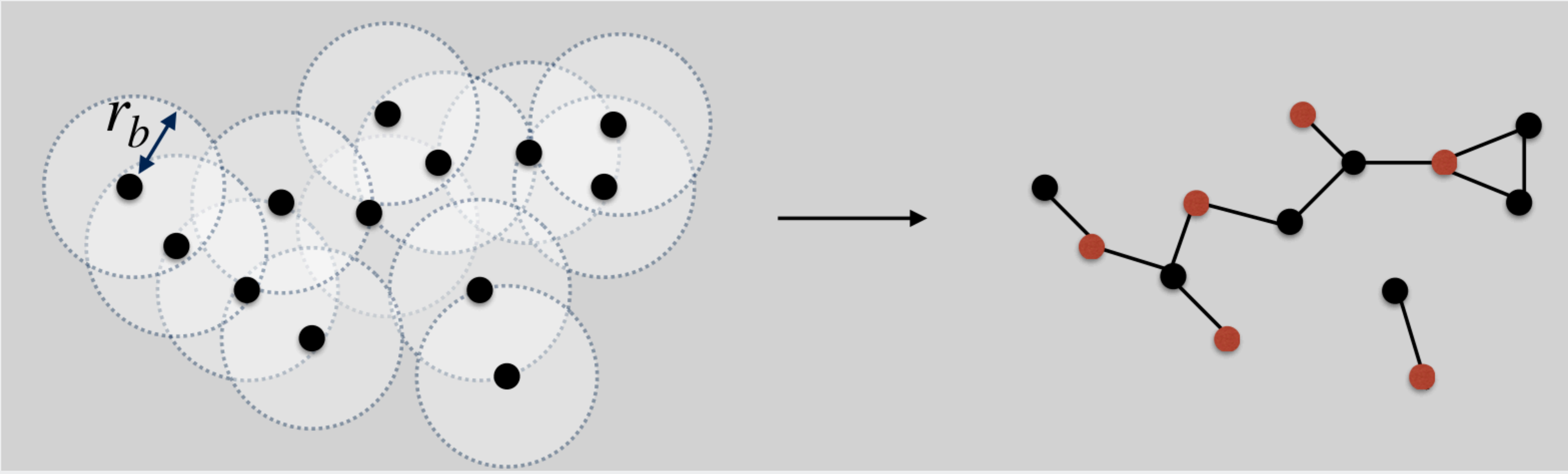




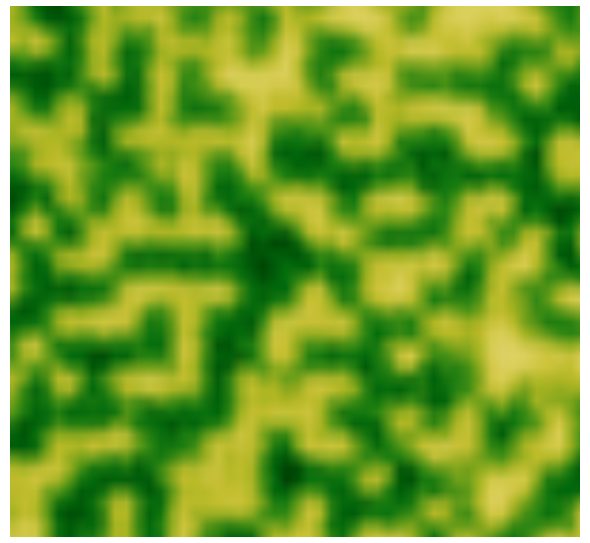
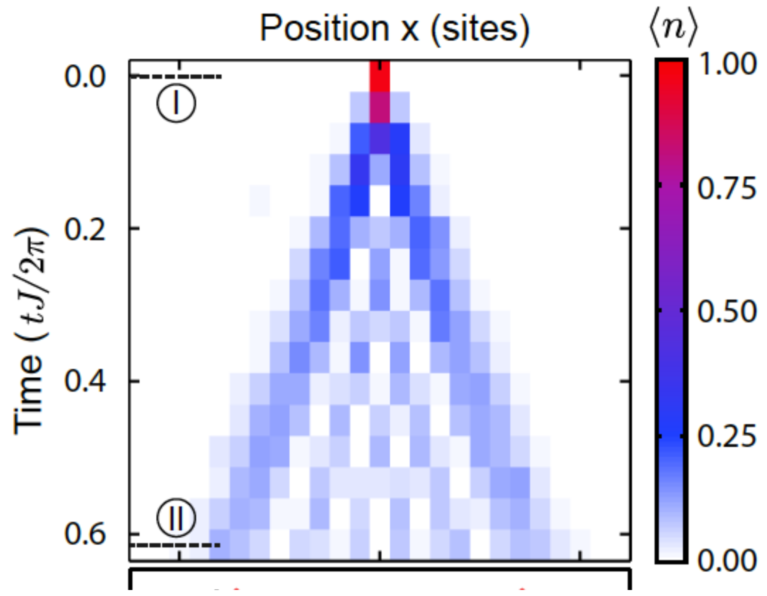
Kasper et al. arXiv:2010.15923

Possible projects (in qiskit-cold-atoms)

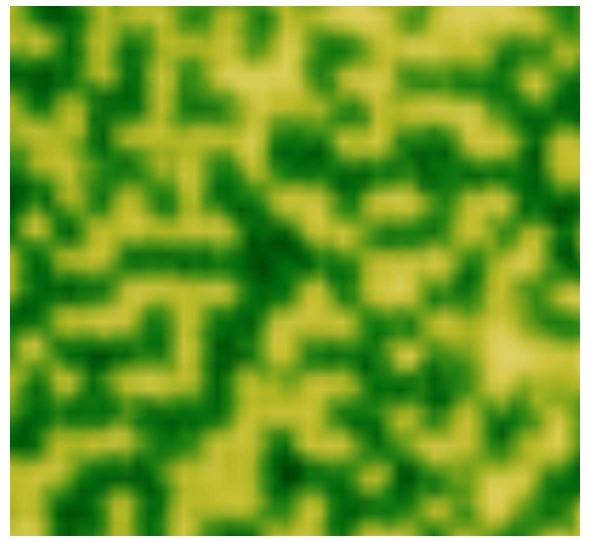
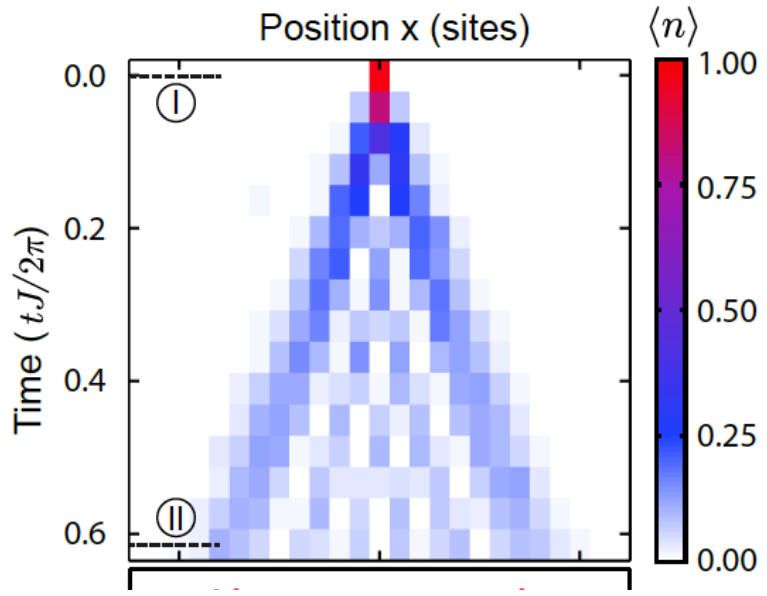
Rydberg atoms for optimization problems

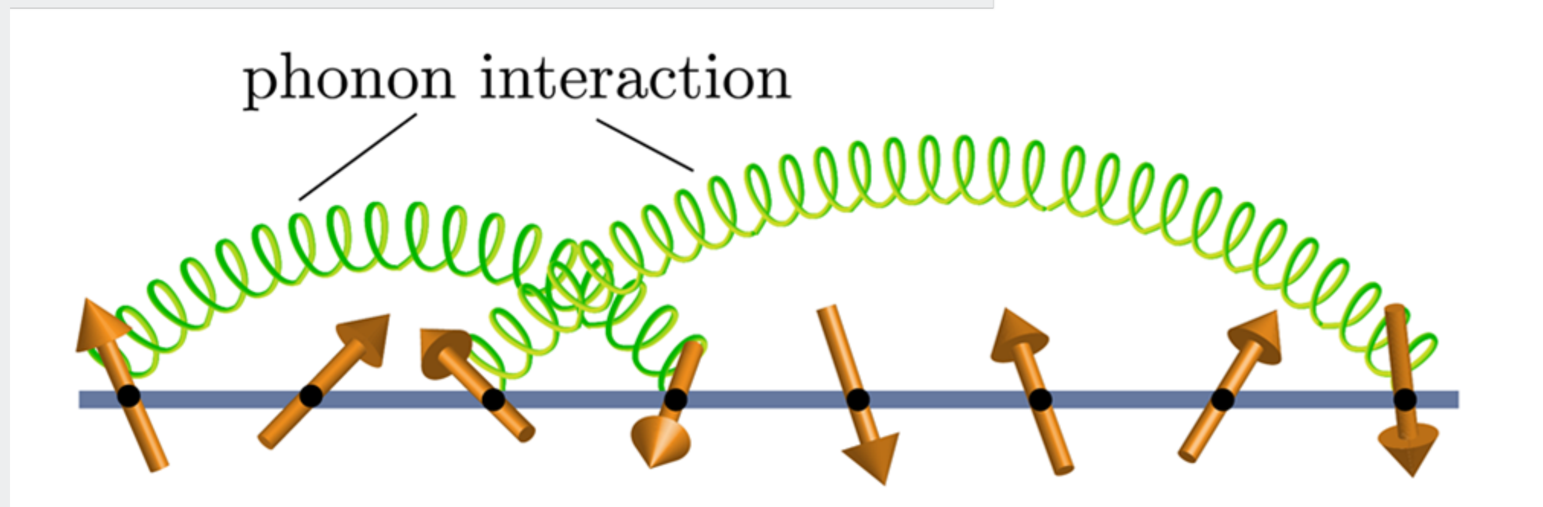
Squeezing on superconducting circuits

Lattice systems for itinerant particles

Universal QC with atomic mixtures



Digital and analog quantum simulators for lattice gauge theories